Data taken with a narrow ($\sigma \approx 0.5\,cm$) electron beam.
In each run the beam impinged at fixed $z$ along a single bar.
Data read with a digital oscilloscope at sampling time $\Delta t = 400\,ps$.

<table>
<thead>
<tr>
<th>Pos.(cm)</th>
<th>36</th>
<th>26</th>
<th>16</th>
<th>6</th>
<th>4</th>
<th>14</th>
<th>24</th>
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<tbody>
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<td>●</td>
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<tr>
<td>19d</td>
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<td>●</td>
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<tr>
<td>2d</td>
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<td>●</td>
<td>●</td>
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<tr>
<td>8dnuda</td>
<td>●</td>
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<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
</tr>
</tbody>
</table>
Analysis topics

- PMT transfert function (laboratory measurement)
- Attenuation length measurements
- Position measurements with $\ln(Q_1/Q_2)$
- Effective velocity $v_{eff}$ measurement
A preliminary measurement is the PMT transfer function. In laboratory PMT output response to a δ pulses (500ps) are sampled with a digital oscilloscope. The easiest approach is to measure the rise time $\tau_r$, the $\tau_f$ and the $fwhm$. A careful analysis leads to a two parameters functions for the PMT transfer function.

$$f(t; \tau_{RC}, n_{RC}) = \frac{1}{\Gamma(n_{RC} + 1)} \left( \frac{t}{\tau_{RC}} \right)^n_{RC} e^{-\frac{t}{\tau_{RC}}}$$

This function gives a good fit the PMT transfer function for different HV.
Attenuation length measurements

The effective bar attenuation length $\lambda_{eff}$ can be estimated using the log of the charge ratio between opposite side PMT. Defining $z$ as the longitudinal coordinate, $L$ the bar length and $G_n$ the gain factor

\[
Q_1 = G_1 E_0 \exp\left(-\frac{L}{2} - z\right) / \lambda_{eff} \\
Q_2 = G_2 E_0 \exp\left(-\frac{L}{2} + z\right) / \lambda_{eff}
\]

\[
\ln\left(\frac{Q_1}{Q_2}\right) = \ln\left(\frac{G_1}{G_2}\right) + z \frac{2}{\lambda_{eff}}
\]

The linear fit is generally as good or better as for bar 19d.

<table>
<thead>
<tr>
<th>Bar</th>
<th>$\lambda_{eff}$ (cm)</th>
<th>Bar</th>
<th>$\lambda_{eff}$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15d</td>
<td>93.96±0.25</td>
<td>16d</td>
<td>79.95±0.17</td>
</tr>
<tr>
<td>17d</td>
<td>72.38±0.13</td>
<td>19d</td>
<td>98.91±0.20</td>
</tr>
<tr>
<td>20</td>
<td>73.57±0.23</td>
<td>2d</td>
<td>100.02±0.31</td>
</tr>
<tr>
<td>8d</td>
<td>70.32±0.80</td>
<td>Ave</td>
<td>84.24±12.0</td>
</tr>
</tbody>
</table>

$\lambda_{eff}$ is not the bulk attenuation length $\lambda = 140 \text{ cm}$ reported in the data sheets. They are related by

\[
\lambda = \frac{\lambda_{eff}}{\langle \cos \Theta \rangle}
\]
where \( < \cos \Theta > = \frac{1 + 1/n_{sc}}{2} = 0.81 \) is averaged over the incident angles.
That gives an average
\[
\overline{\lambda} = \frac{\lambda_{eff}}{< \cos \Theta >} = 103.49 \pm 14.74 cm
\]
There is a significant different from the expected value and there is a spread difficult to understand.
Influence of reflection loss on $\lambda$

An effect not yet included neither in MC nor in analysis is the reflection loss at the surface. Defining $R_2$ the reflection efficiency below the critical angle and $a$ the bar thickness, the number of reflections for photons travelling at angle $\Theta$ for a distance $x$ is

$$N_R = \frac{x}{a \tan \Theta}$$

This effect gives a reflection absorption length as

$$\lambda_R = \frac{a}{< \tan \Theta > \ln \frac{1}{R_2}}$$

Therefore $\lambda_{eff}$ can be defined as

$$\frac{1}{\lambda_{eff}} = \frac{1}{\lambda} \frac{1}{< \cos \Theta >} + \frac{1}{\lambda_R}$$

If $R_2 = 1 - \epsilon$ for $\epsilon << 1$

$$\lambda_R = \frac{a}{< \tan \Theta > \epsilon}$$

**Estimating $R_2$ from $\lambda_{eff}$ measurements**

<table>
<thead>
<tr>
<th>Bar</th>
<th>15d</th>
<th>16d</th>
<th>19d</th>
<th>20</th>
<th>2d</th>
<th>8d</th>
<th>Ave</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.60</td>
<td>1.21</td>
<td>1.63</td>
<td>0.43</td>
<td>1.56</td>
<td>0.40</td>
<td>1.76</td>
</tr>
</tbody>
</table>

If the difference between $\lambda_{eff}$ and the bulk value is due to reflection losses, $\epsilon$ can be obtained for each bars. A spread in $\epsilon$ that depends on the surface quality is more credible that the same spread in bulk property.
Position measurements

Using $\lambda_{eff}$ we can obtain a position measurement from

$$\ln(Q_1/Q_2) = \ln(G_1/G_2)_{fit} + z\frac{2}{\lambda_{eff,fit}}$$

Averaging all measurements on a bar, the position resolution is $\sigma(z) = 2.6cm$.

This error does not depend on the hit position except when its distance is comparable to PMT diameter.

In this case the linear relation between $\ln(Q_1/Q_2)$ and $z$ breaks down because photons at $|\cos \Theta| < \frac{1}{n_{sc}}$ can reach the PMT without reflection.
Effective velocity $v_{eff}$ measurements

The formulae for the timing are

\[ t_1 = t_0 + \left( \frac{L}{2} - z \right) \frac{1}{v_{eff}} \]
\[ t_2 = t_0 + \left( \frac{L}{2} + z \right) \frac{1}{v_{eff}} \]
\[ t_2 - t_1 = \frac{2z}{v_{eff}} \]

Important: the timing $t_1, t_2$ depends on the algorithm.
We used amplitude normalized threshold, the timing fires when the signal cross $\alpha\%$ of the maximum amplitude: 10\%, 50\%, 90\%.
The time profile of the signal is due to different components:

- Transfer function of the PMT
- Photon time distribution due to the scintillation process
- Photon propagation in the bar

The last contribution depends on hit position, that implies that $v_{eff}$ depends on the fraction of photons $\epsilon$ contributing to the timing.
Different $\epsilon$ corresponds approximately to different $\alpha$.
Different $\epsilon$ correspond to different $\cos(\Theta_\epsilon)$, where $\cos(\Theta_\epsilon)$ is the angle within which the fraction $\epsilon$ is emitted.
\[(1 - \cos(\Theta_\epsilon)) = (1 - \frac{1}{n_{sc}})\epsilon\]

\(v_{\text{eff}}\) is established by the formulae

\[
t(x, \cos(\Theta)) = \frac{z}{\cos(\Theta)} \frac{n_{sc}}{c}
\]

\[
t_\epsilon(x) = t(x, \cos(\Theta_\epsilon)) = \frac{z}{\cos(\Theta_\epsilon)} \frac{n_{sc}}{c}
\]

\[
v_{\text{eff},\epsilon} = \frac{x}{t_\epsilon(x)} = \frac{c}{n_{sc}} \cos(\Theta_\epsilon) = \frac{c}{n_{sc}} (1 - (1 - \frac{1}{n_{sc}})\epsilon)
\]

Therefore for \(\epsilon \to 0\) (\(\alpha \to 0\)) \(v_{\text{eff},0} \to \frac{c}{n_{sc}} = 18.87 \frac{cm}{ns}\).

For \(\epsilon \to 1\) (\(\alpha \to 1\)) \(v_{\text{eff},0} \to \frac{c}{n_{sc}^2} = 11.87 \frac{cm}{ns}\).

These two values constrain the range of values of \(v_{\text{eff}}\).

In the test beam the relation between \(z\) and \(t_2 - t_1\) is measured for several bars and it is very linear.

<table>
<thead>
<tr>
<th>Bar</th>
<th>(v_{\text{eff},0.1} \frac{cm}{ns})</th>
<th>(v_{\text{eff},0.5} \frac{cm}{ns})</th>
<th>(v_{\text{eff},0.9} \frac{cm}{ns})</th>
</tr>
</thead>
<tbody>
<tr>
<td>15d</td>
<td>14.81±0.05</td>
<td>14.17±0.02</td>
<td>13.76±0.05</td>
</tr>
<tr>
<td>16d</td>
<td>15.03±0.04</td>
<td>14.31±0.02</td>
<td>13.85±0.03</td>
</tr>
<tr>
<td>17d</td>
<td>15.42±0.02</td>
<td>14.62±0.02</td>
<td>14.31±0.02</td>
</tr>
<tr>
<td>19d</td>
<td>14.98±0.02</td>
<td>14.24±0.01</td>
<td>13.82±0.02</td>
</tr>
<tr>
<td>20</td>
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<td>14.38±0.02</td>
<td>13.62±0.04</td>
</tr>
<tr>
<td>2d</td>
<td>15.13±0.04</td>
<td>14.33±0.03</td>
<td>14.06±0.05</td>
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<tr>
<td>8d</td>
<td>15.06±0.33</td>
<td>14.33±0.25</td>
<td>13.82±0.21</td>
</tr>
</tbody>
</table>

The trend of increasing \(v_{\text{eff},\alpha}\) with decreasing \(\alpha\) is confirmed.