A study on various gain calibration methods of the photo-sensor, SiPM

光検出器SiPMのゲイン較正手法に関する研究

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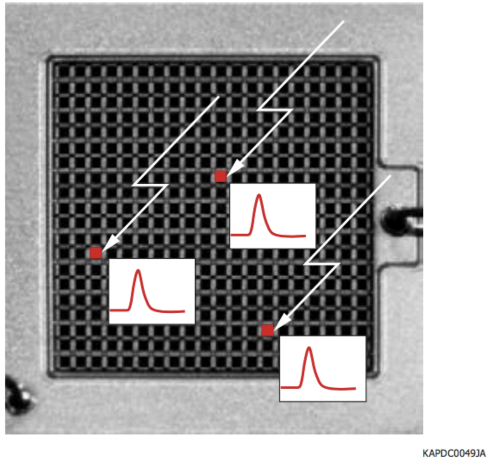
Outline

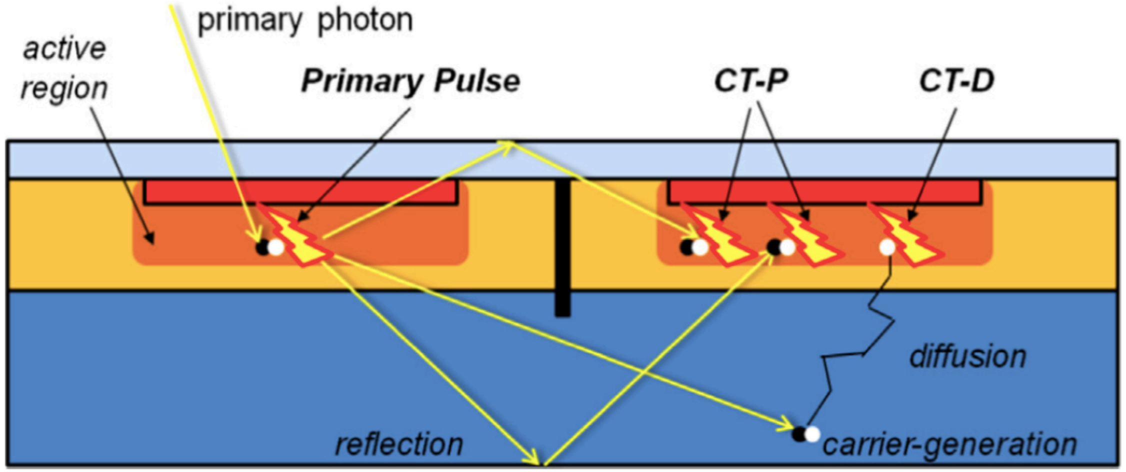
- ➢Introduction: About SiPM
- ➢Statistical Method
- ➢Waveform Method

SiPM

- SiPM is photon detector made of semiconductor.
  - Photon generates electron & hole pair.
  - multiplication of electron by avalanche
  - same output from a cell (Geiger mode) → photoncounting
- Advantage
  - small
  - low bias voltage
  - can be used in B field
- Noise
  - Dark Count
  - Crosstalk
  - Afterpulsing



Hamamatsu 光半導体素子ハンドブック



Ferenc Nagy et al. ”Afterpulse and delayed crosstalk analysis on a STMicroelectronics silicon photomultiplier”,Nuclear Instruments and Methods in Physics Research A 759 (2014) 44 – 49

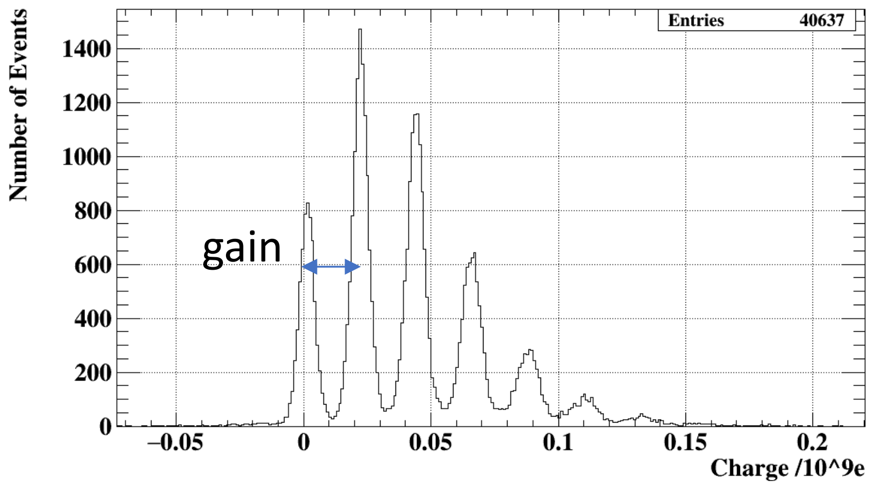
Gain

- Gain: relation b/w output & # of photoelectrons
  - depends on over voltage, temperature, …
  - must be checked regularly in experiment

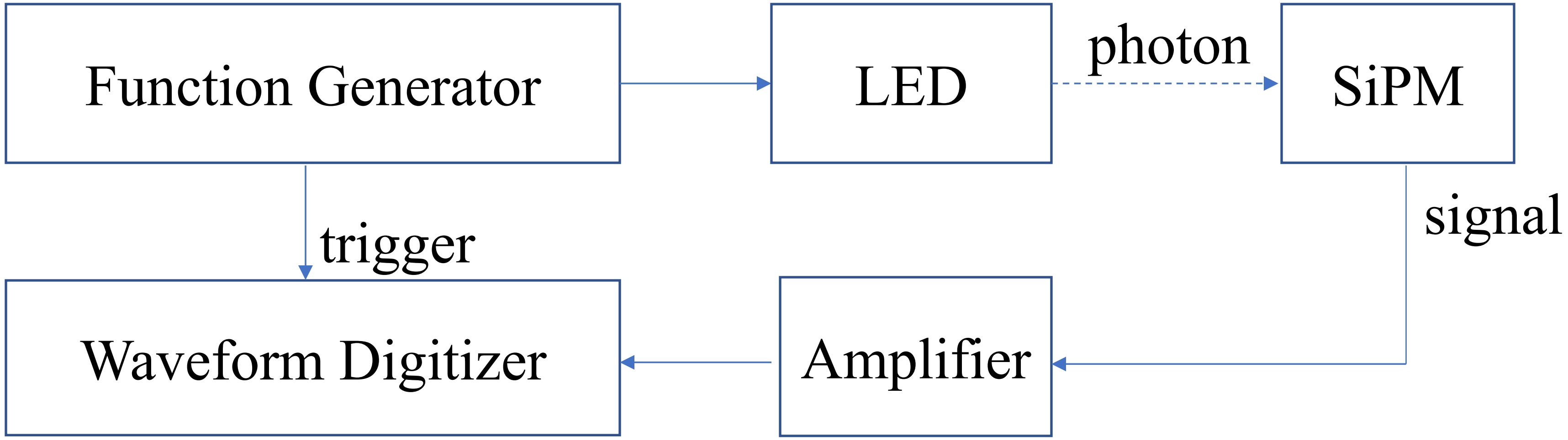
- Measuring Gain from single p.e. (photoelectron)
  - Since SiPM can detect 1 photon, output charge distribution has peak structure.
    → Gain can be directly obtained from interval of peaks
  - Limitation
    - need good S/N to separate each peak
    - need dedicated run if dark count rate is small

  → We will introduce redundant methods

Charge Histogram



Set up



- SiPM : MPPC S13360-3050PE (Hamamatsu)
  - effective area : 3 mm × 3 mm
  - pixel pitch : 50 μm
  - # of pixels : 3600
- Amplifier : MAR amplifier (PSI) (amplification factor: ~70)
- Waveform Digitizer : DRS4 chip (PSI)
  - sampling rate : 1.6 GHz
- Temperature was kept at 23 ℃

Outline

- ➢Introduction: About SiPM
- ➢Statistical Method
- ➢Waveform Method

Previous Study: PMT

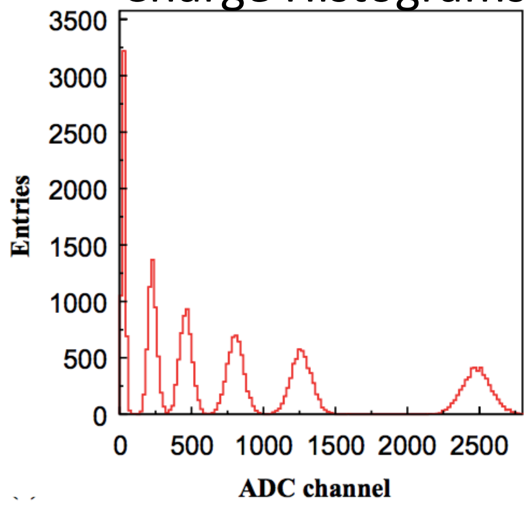
A. Baldini et al., MEG Collaboration,
Nucl. Instr. Methods Phys. Res. Sect. A 545 (2005) 753

- When # of photoelectrons follows Poisson distribution (mean: μ), mean & variance of output charge are
  - $mean = gain \cdot \mu$
  - $variance = gain^2 \cdot \mu$

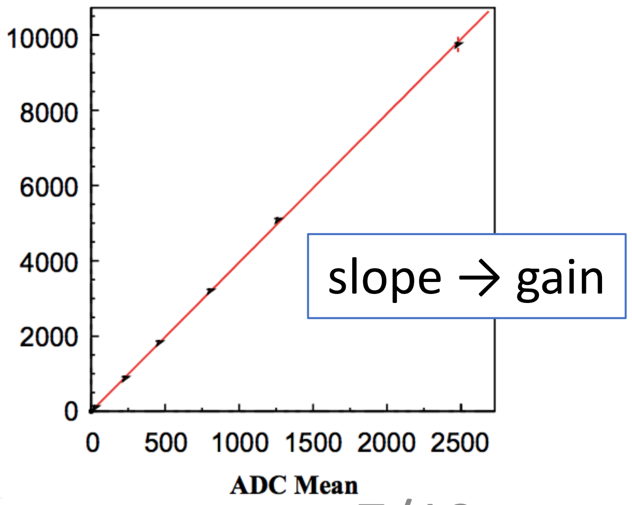
$$\boxed{gain = \frac{variance}{mean}}$$

- Charge distribution is broadened by Gaussian noise (σ).
  $\rightarrow$ subtract σ^2 from variance
- already used in experiments (ex. MEG)

Charge Histograms



variance vs mean



Previous Study: SiPM

Chmill, V. *et al.* Nucl.Instrum.Meth. A854 (2017)

- Assuming # of crosstalks from 1 cell follows Poisson distribution, total # of p.e. follows Generalized Poisson distribution.

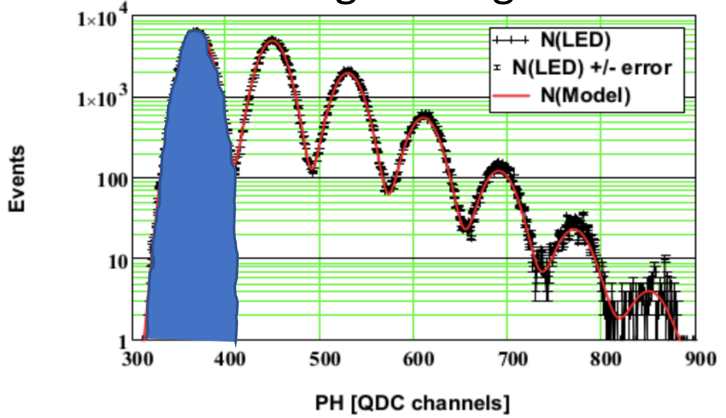
$$GP_{\mu,\lambda}(k) := \frac{\mu(\mu+k\lambda)^{k-1}e^{-(\mu+k\lambda)}}{k!}$$

μ: mean w/o crosstalk
λ: crosstalk probability

  - $E[k] = \frac{\mu}{1-\lambda}$
  - $V[k] = \frac{\mu}{(1-\lambda)^3}$

※ when each of crosstalk & after-pulse probability < 25 %

Charge Histogram



- Then, defining Excess Noise Factor as $ENF := \mu \frac{variance}{mean^2}$,

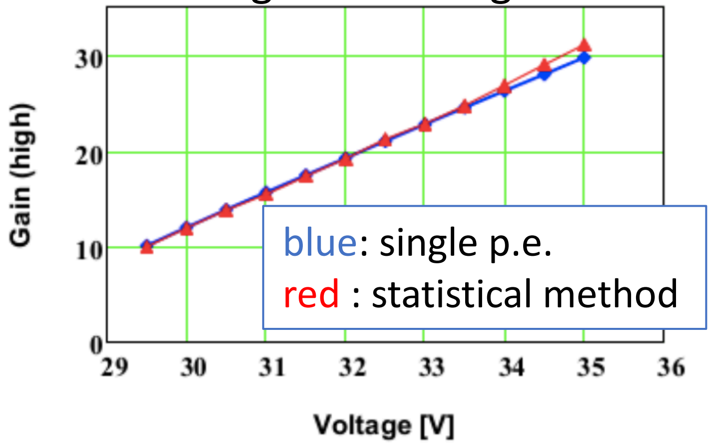
$$\boxed{gain = \frac{1}{ENF^2}\frac{variance}{mean}}$$

- ENF is calculated from pedestal fraction.

$$GP_{\mu,\lambda}(k=0) = e^{-\mu}$$

- tested with KETEK SiPM (4384 pixels, 15μm ×15μm area)

gain vs voltage



blue: single p.e.
red : statistical method

Characteristics of the Method

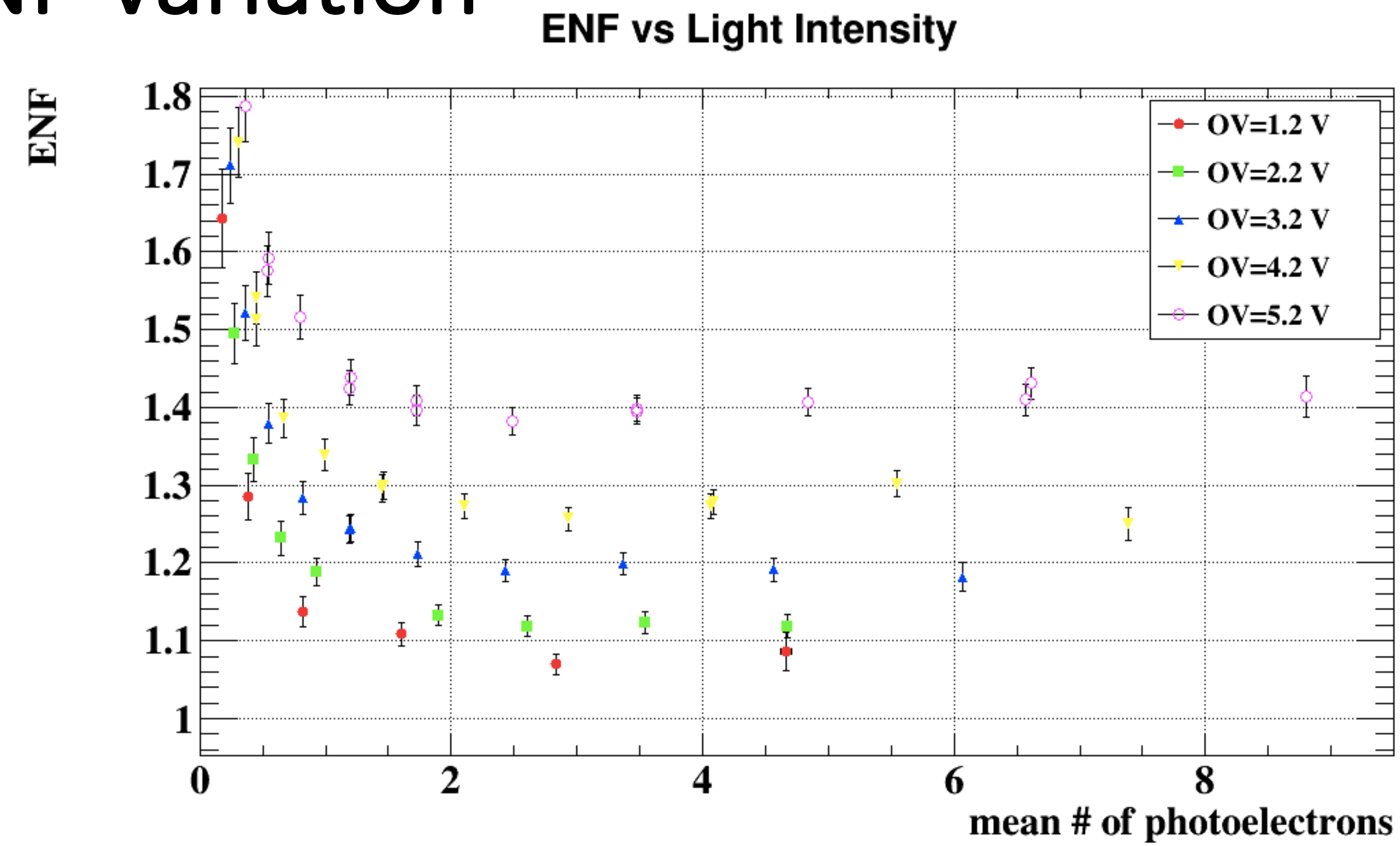
- ➢Advantage
  - ocan be used even when peaks are not separated
    - when S/N is bad
    - when using strong light
- ➢Limitation
  - oConstant light source is necessary.
  - oneed to know or assume ENF beforehand

We tried this method with Hamamatsu MPPC.

→ Some problems were found.

- ENF depends on light intensity
- variance/mean depends on light intensity

ENF Variation



$$ENF := \mu \frac{variance}{mean^2}$$

$$gain = \frac{1}{ENF^2} \frac{variance}{mean}$$

<u>Over Voltage</u>

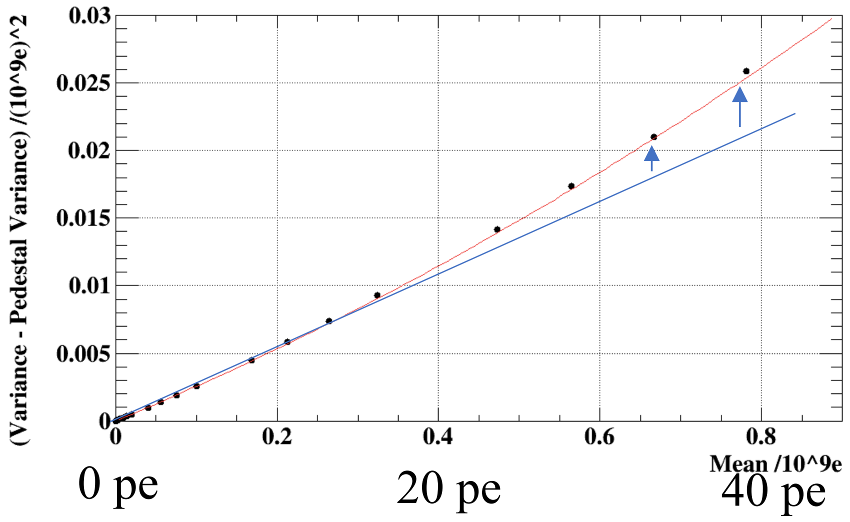
- large ENF when over voltage is large
  → can be explained by correlated noises

<u>Light Intensity</u>

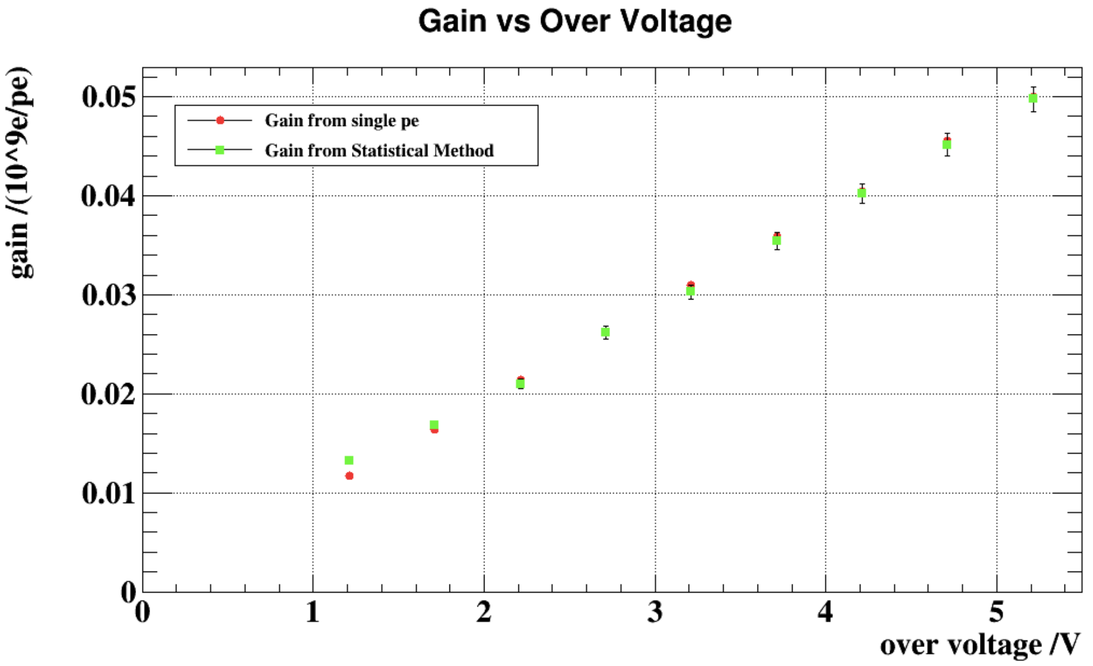
- large ENF when using too weak light
- converge in strong light region
  → dark count contribution in weak light region?

Non-linearity of variance vs mean

variance vs mean @OV=2.2 V



measure at various over voltage



- Plot of variance vs mean is not linear.
- because of improper assumption?
  - crosstalk/after-pulse probability, delayed crosstalk
  - model of correlated noises

→ fit by quadratic and use slope @origin

- ENF was calculated with light whose mean p.e. is ~ 4.
- precision: ~ 2.5 %
- Consistent gain was obtained.

Summary

➢Summary

- Previous study shows gain can be obtained from statistics of charge distribution.
- We tested the method with Hamamatsu MPPC.
- Although it was found that ENF & variance/mean depend on light intensity, we obtained gain consistent with that from single p.e.

➢Problems & to do

- The reason of
  - large ENF @weak light
  - non-linearity b/w variance & mean

  are not understood.

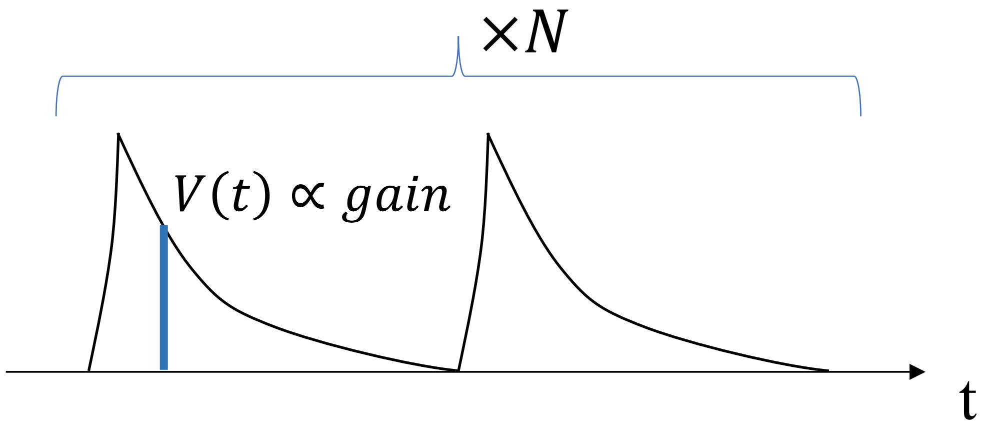
  → investigate using simulation

Outline

- Introduction: About SiPM
- Statistical Method
- Waveform Method

Previous Study

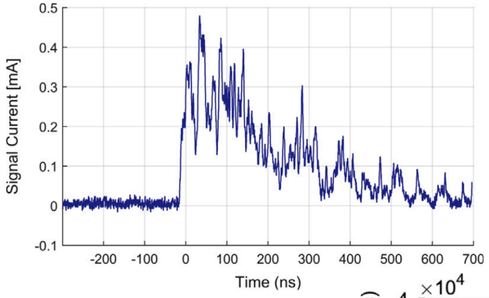
J. Stein *et al.* Nuclear Instruments and Methods in Physics Research A 782 (2015)



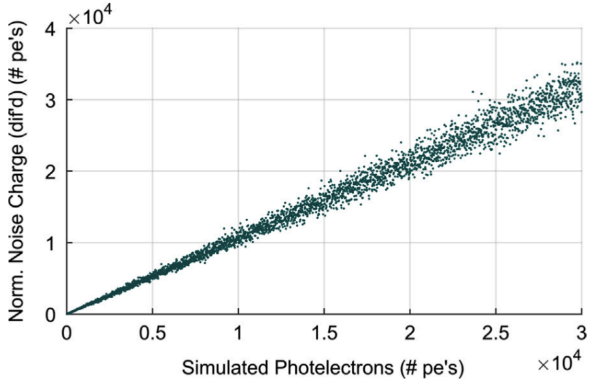
- When pulses are separated,
  - $charge = \int V(t)\,dt \propto N_{pulse} \cdot gain$
  - $height\ squared = \int (V(t))^2\,dt \propto N_{pulse} \cdot gain^2$

→ $\boxed{gain \propto heigh\ squared/charge}$

  - You can monitor gain without any dedicated run when waveform is accessible.
  - Constant light source is not needed.



- to separate pulses, take derivatives
  - Considering S/N, 2nd derivative is best.
- has been demonstrated using PMT (sampling rate: 2.5 GHz)



- We applied this method to SiPM for the first time.

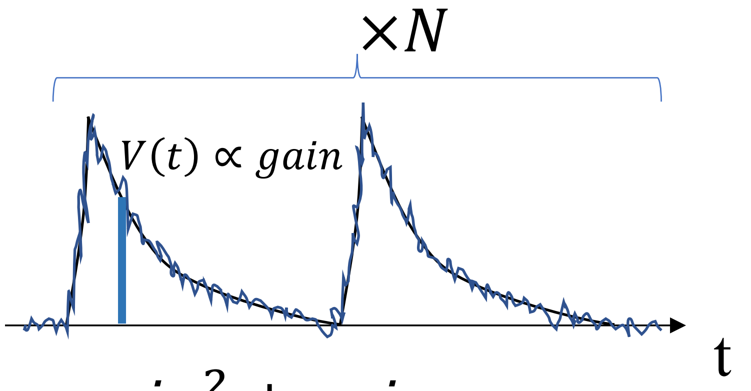
→ effect of overlapping due to prompt crosstalk

Extension of the model

○effect of white noise $V_{noise}(t)$

- $charge = \int\left(V(t) + V_{noise}(t)\right) dt \propto N_{pulse} \cdot gain$
- $height\ squared = \int\left(V(t) + V_{noise}(t)\right)^2 dt \simeq const. \cdot N_{pulse} \cdot gain^2 + noise$

→ $\boxed{height\ squared = c_0 + c_1 \cdot charge\ ,\ c_1 \propto gain}$



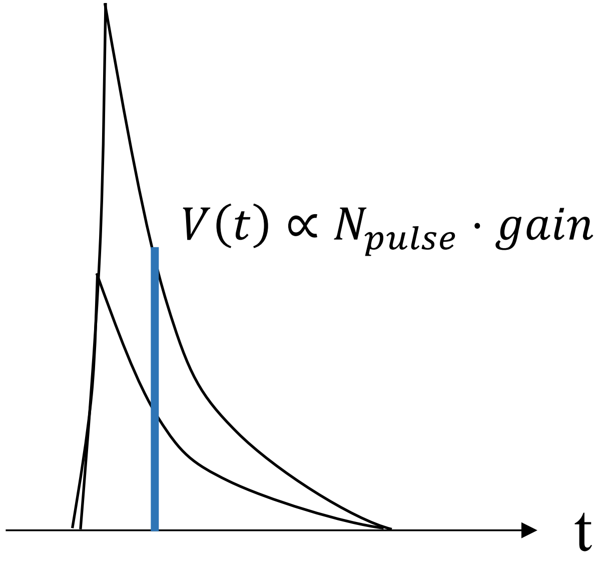
○effect of overlapping

- due to large # of photoelectrons in unit time
- due to prompt crosstalk

When completely overlapped,

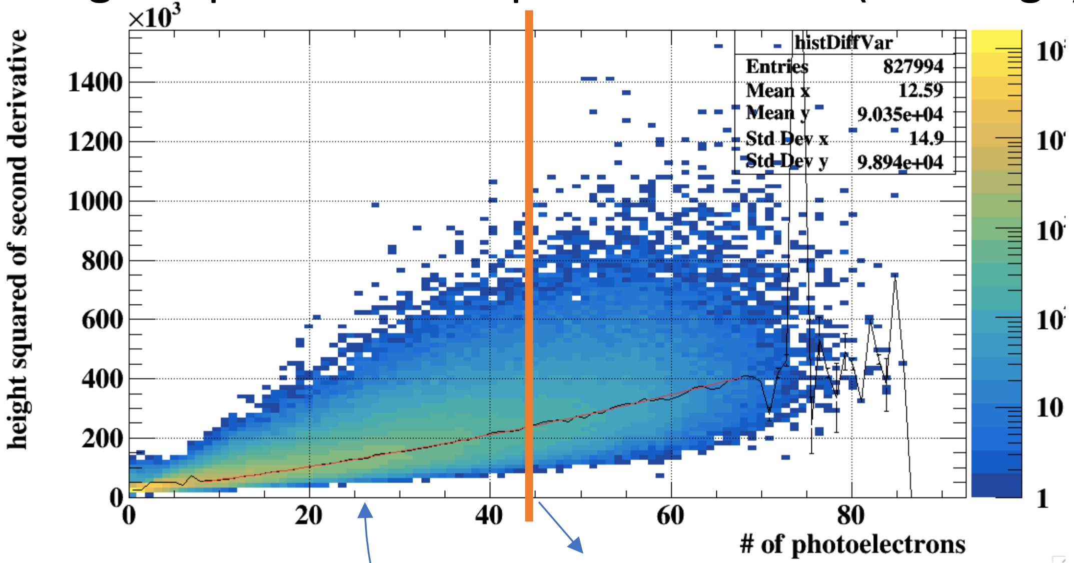
- $height\ squared = \int\left(V(t)\right)^2 dt \propto N_{pulse}^2 \cdot gain^2$

→ Overlapping results in large height squared.



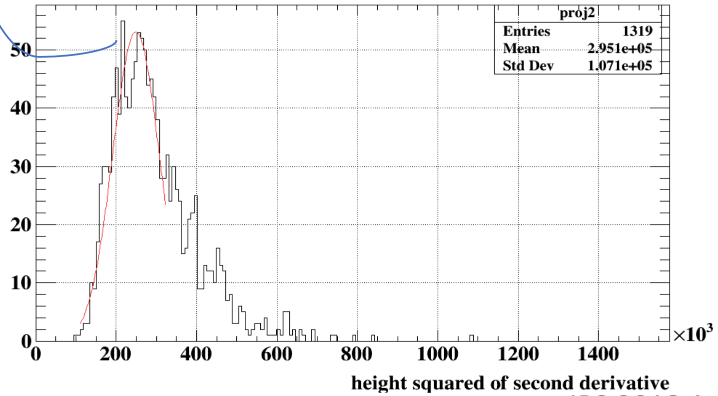
Result

height squared vs # of photoelectrons (∝ Charge)



Height Squared Histogram

peak is drawn as line

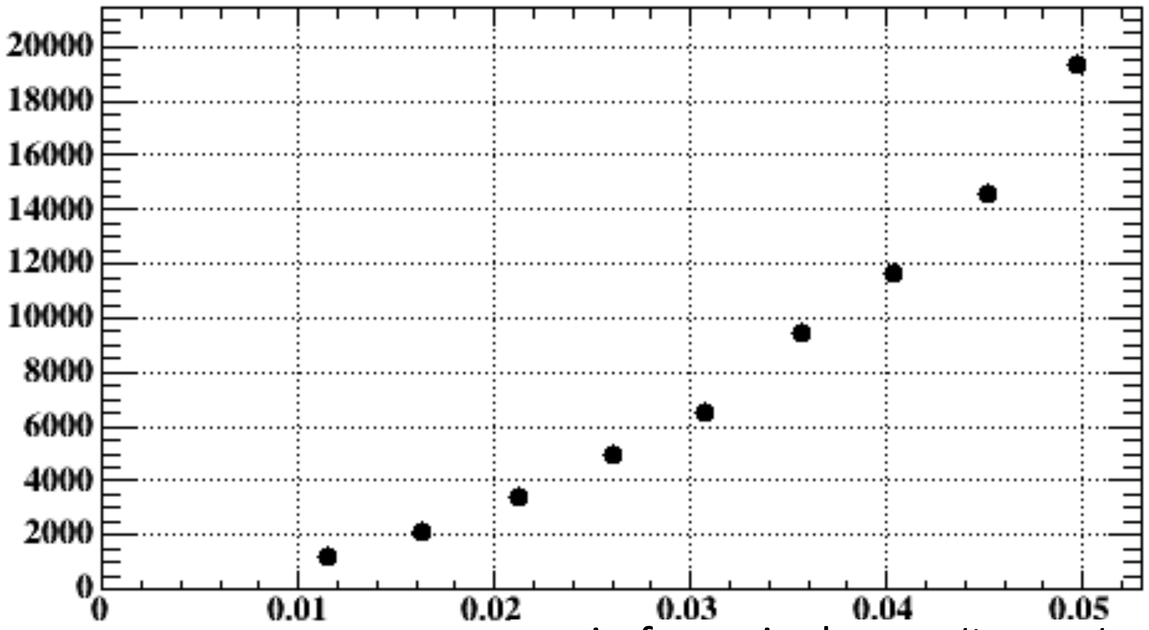


- offset
  - effect of white noise
- tail
  - effect of overlap due to crosstalk
  - search for peak @ each # of photoelectron
- peak shift
  - deviate from linear in large # of photoelectrons
  - Large # of photoelectrons in unit time makes much overlapping.

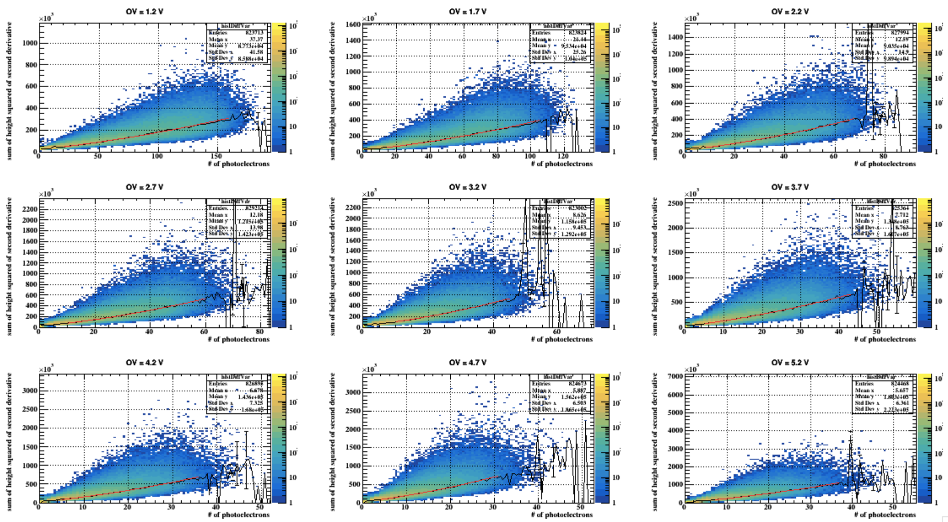
Result

➢ Measurement changing Over Voltage

slope @origin



gain from single p.e. /(10^9e/pe)



- Correlation is not linear, but clear enough to monitor gain.
- If you calibrate beforehand, you can also obtain absolute gain.

Summary

➢Summary

- Previous study shows PMT gain can be monitored using waveform.
- We considered effect of noise and overlap.
- We obtained correlation clear enough to monitor gain.

➢Problems & to do

- Stability of correlation must be checked.
  - noise
  - individual difference
- should be checked using other light source such as scintillator
- Effect of sampling rate should be checked.

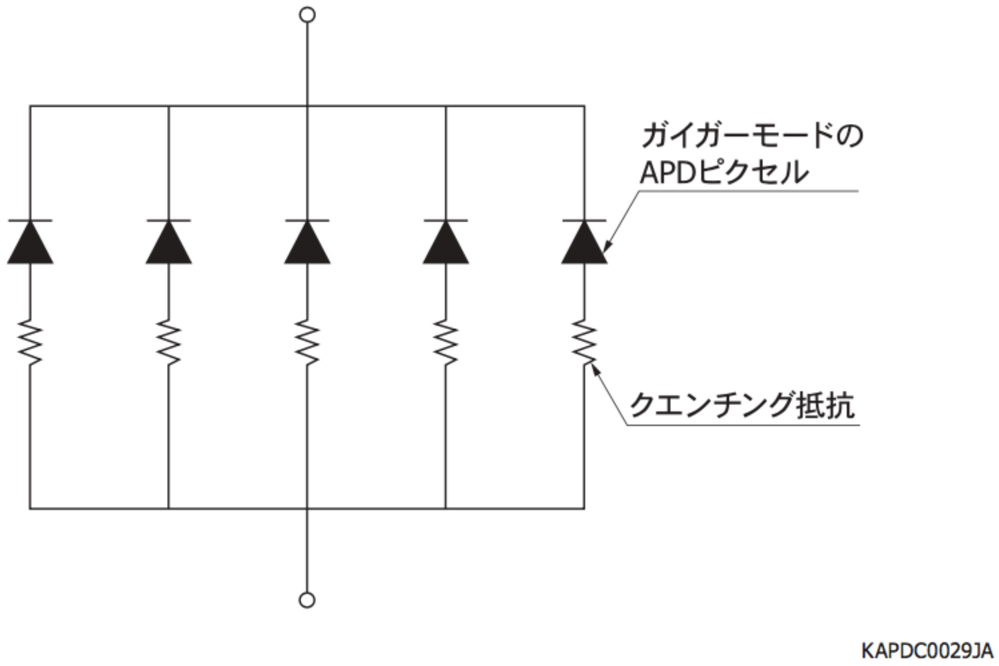
Summary as Comparative Study

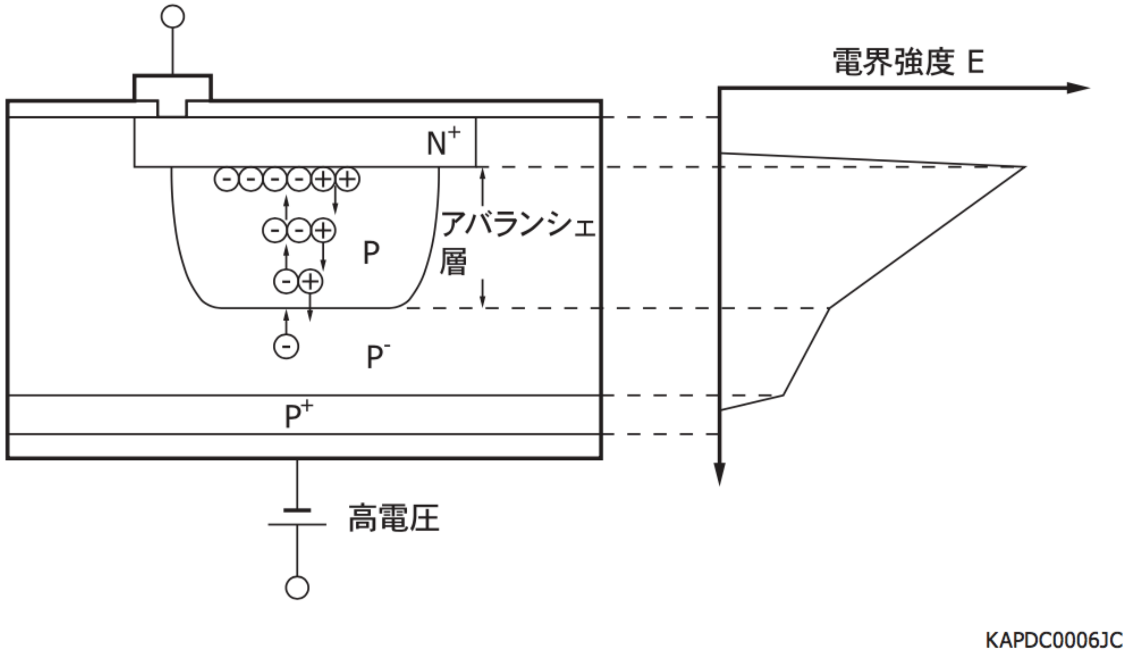
- Single p.e.
  - measure single p.e. directly
    - peaks must be separated
- Statistical Method
  - use mean & variance
    - advantage: S/N is not required
    - limitation : need constant light source, assumption on ENF
- Waveform Method
  - use waveform
    - advantage: w/o dedicated run
    - limitation: need calibration to obtain gain itself

- To understand behavior of each variable, simulation study will be continued.

Back up

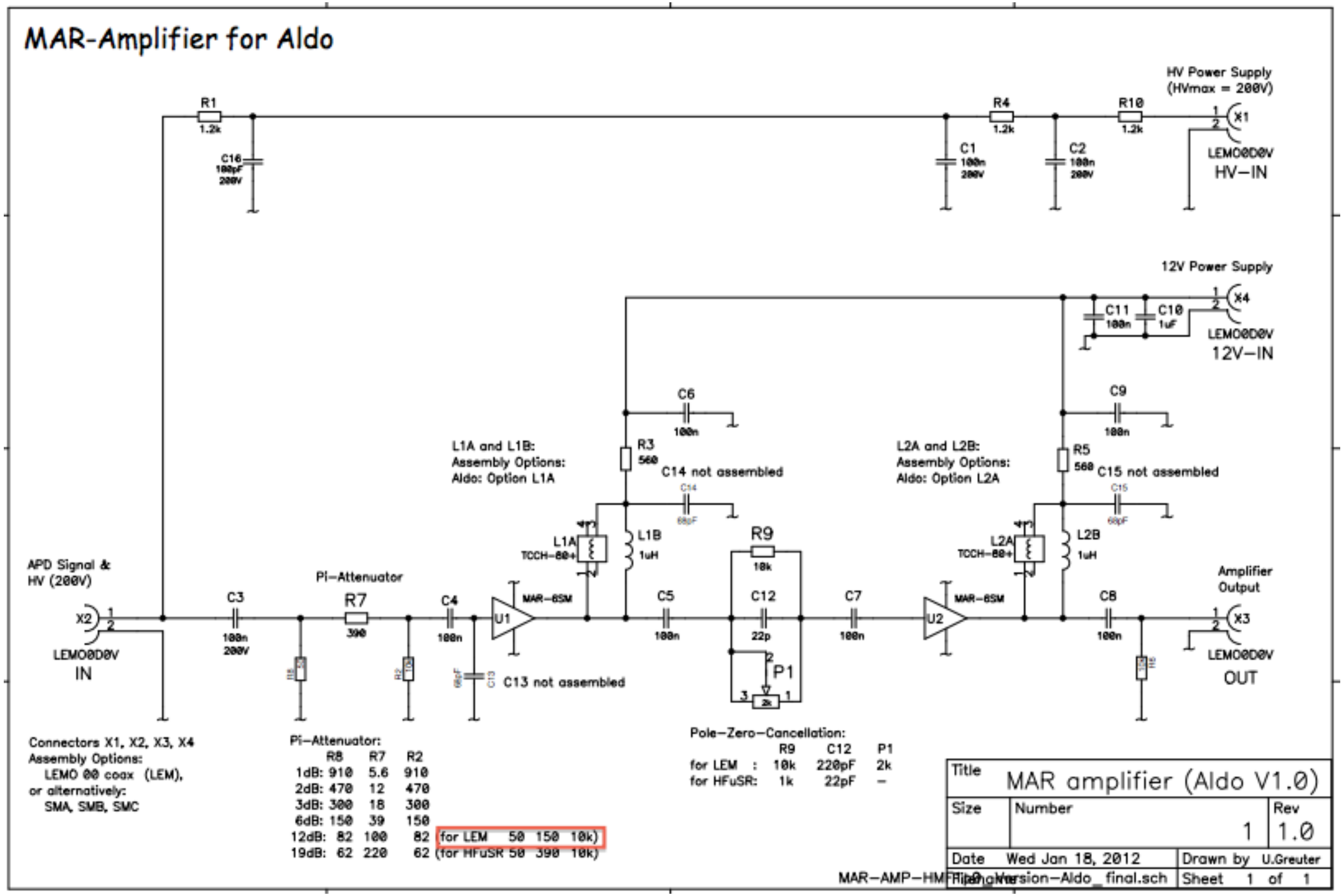
SiPM



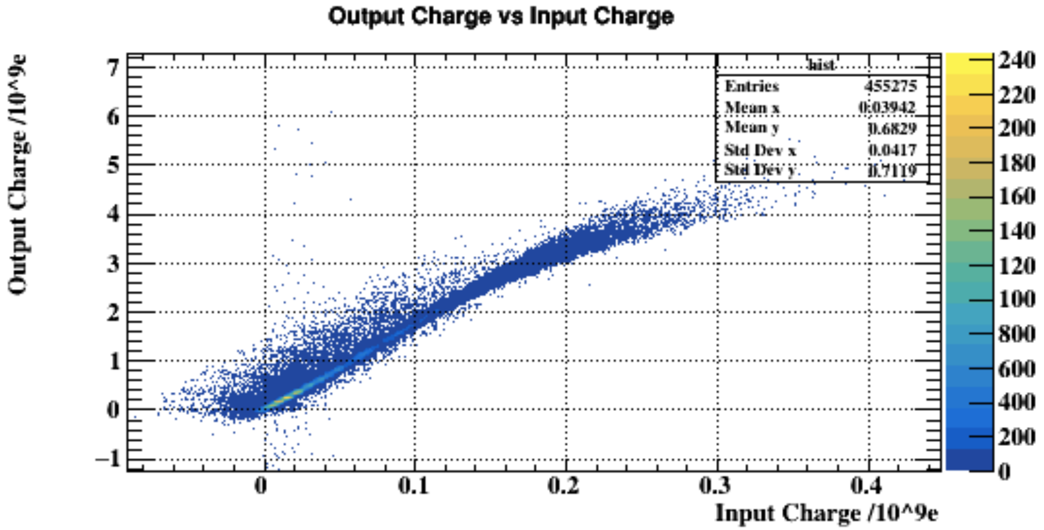


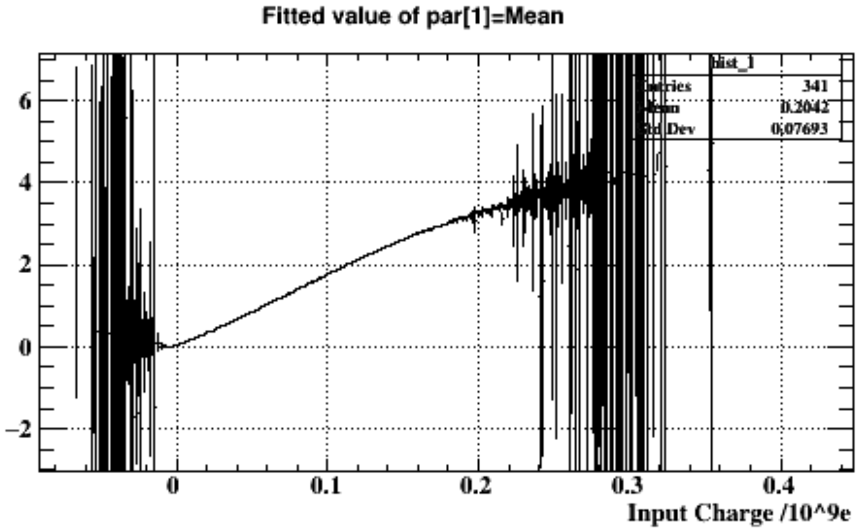
Hamamatsu 光半導体素子ハンドブック

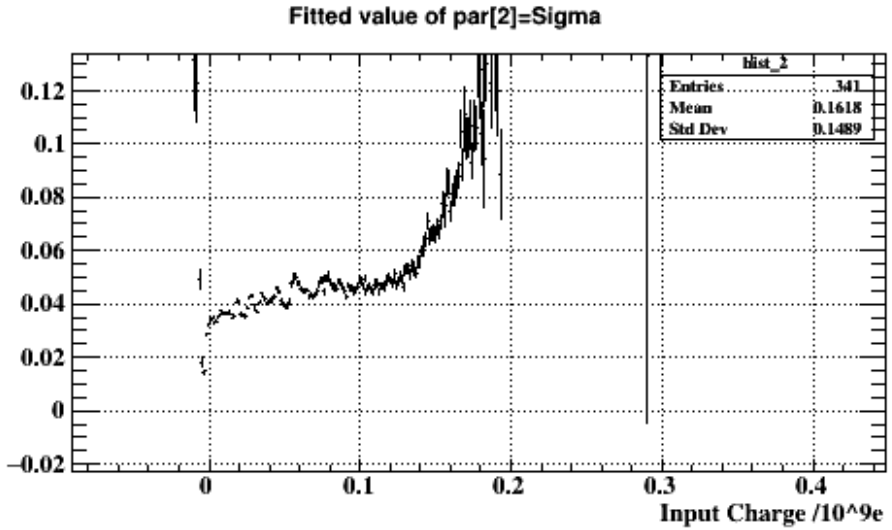
PSI amp


MAR-Amplifier for Aldo
HV Power Supply
(HVmax = 200V)
R1
1.2k
C16
100pF
200V
R4
1.2k
R10
1.2k
X1
C1
100n
200V
C2
100n
200V
LEMO0D0V
HV-IN
12V Power Supply
X4
C11
100n
C10
1uF
LEMO0D0V
12V-IN
C6
100n
C9
100n
R3
560
C14 not assembled
R5
560
C15 not assembled
L1A and L1B:
Assembly Options:
Aldo: Option L1A
L2A and L2B:
Assembly Options:
Aldo: Option L2A
L1A
TCCH-80+
L1B
1uH
R9
10k
L2A
TCCH-80+
L2B
1uH
APD Signal &
HV (200V)
Pi-Attenuator
R7
390
C3
100n
200V
C4
100n
U1
MAR-6SM
C5
100n
C12
22p
C7
100n
U2
MAR-6SM
C8
100n
Amplifier
Output
X2
LEMO0D0V
IN
X3
LEMO0D0V
OUT
P1
2k
C13 not assembled
Connectors X1, X2, X3, X4
Assembly Options:
LEMO 00 coax (LEM),
or alternatively:
SMA, SMB, SMC
Pi-Attenuator:
R8 R7 R2
1dB: 910 5.6 910
2dB: 470 12 470
3dB: 300 18 300
6dB: 150 39 150
12dB: 82 100 82 (for LEM 50 150 10k)
19dB: 62 220 62 (for HFuSR 50 390 10k)
Pole-Zero-Cancellation:
R9 C12 P1
for LEM : 10k 220pF 2k
for HFuSR: 1k 22pF -
Title MAR amplifier (Aldo V1.0)
Size
Number 1
Rev 1.0
Date Wed Jan 18, 2012
Drawn by U.Greuter
MAR-AMP-HMF
Filename
Version-Aldo_final.sch
Sheet 1 of 1

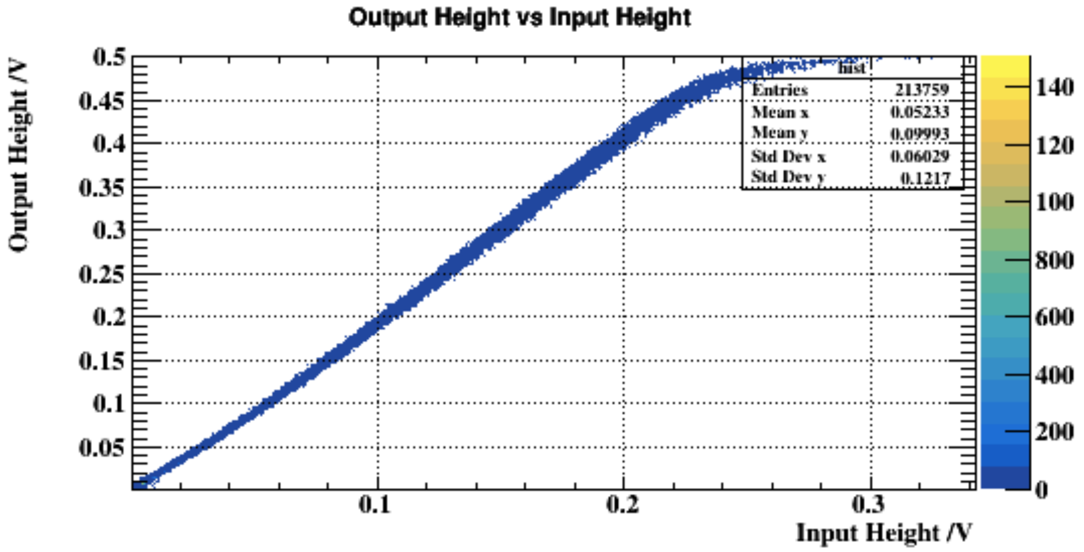
Amp Linearity (Charge)

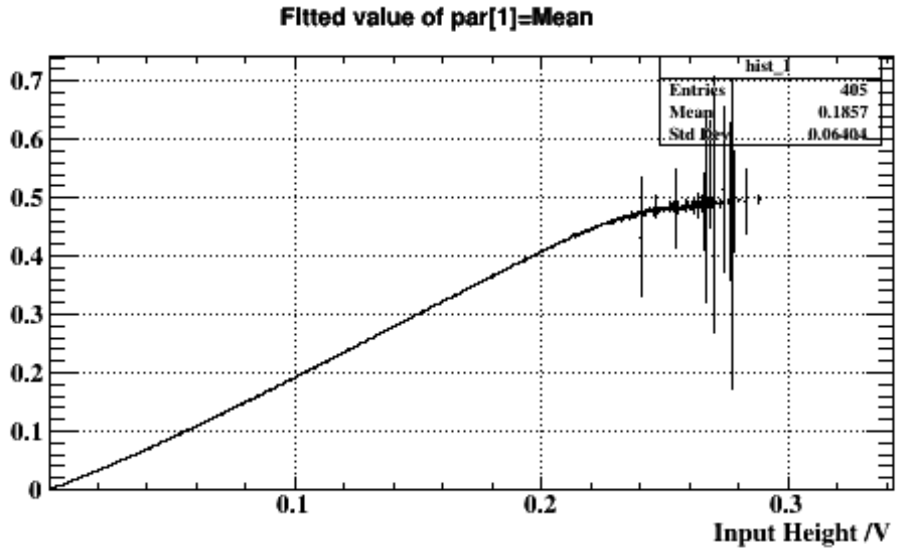


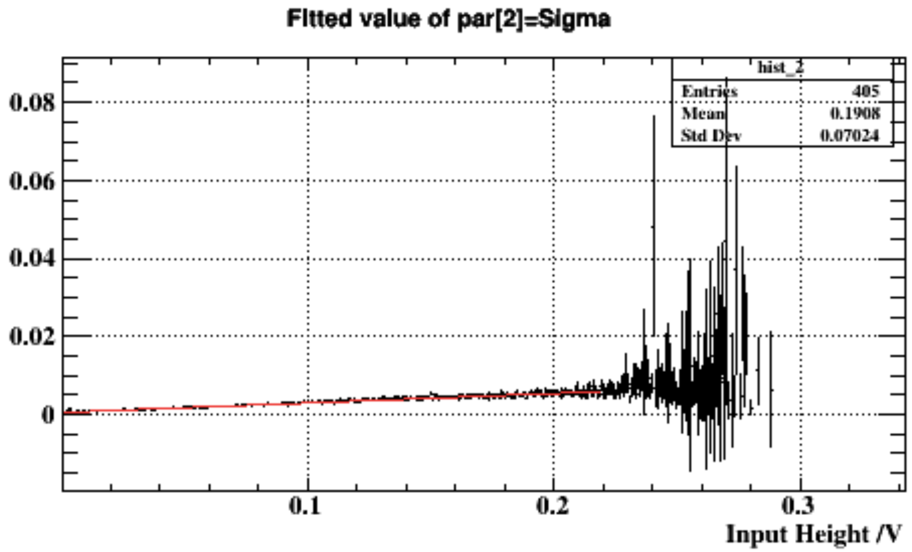




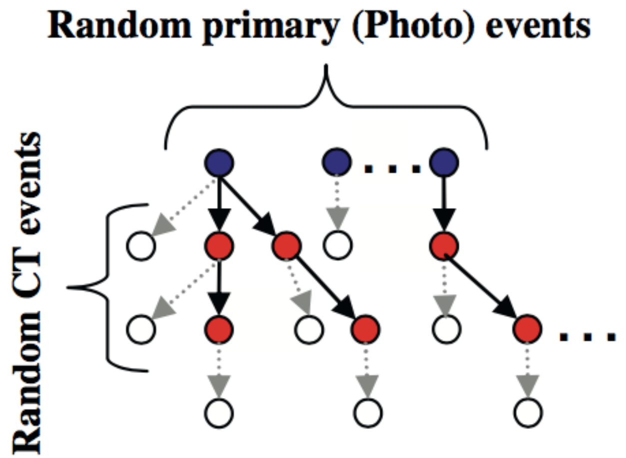
Amp Linearity (Height)







Charge distribution



- Generalized Poisson distribution

$$GP_{k;\mu,\lambda} = \frac{\mu(\mu+k\lambda)^{k-1}e^{-(\mu+k\lambda)}}{k!}$$

  - $mean = \frac{\mu}{1-\lambda}$
  - $variance = \frac{\mu}{(1-\lambda)^3}$

- broaden following Gaussian distribution

$$Gauss_{x;k,\sigma_k} = \frac{1}{\sqrt{2\pi}\sigma_k}\exp\left(-\frac{(x-x_k)^2}{2\sigma_k^2}\right)$$

  - charge of k electron: $x_k = pedestal + gain \cdot k$
  - standard deviation of kth peak: $\sigma_k^2 = \sigma_0^2 + \sigma_c^2 \cdot k$

- Convolution

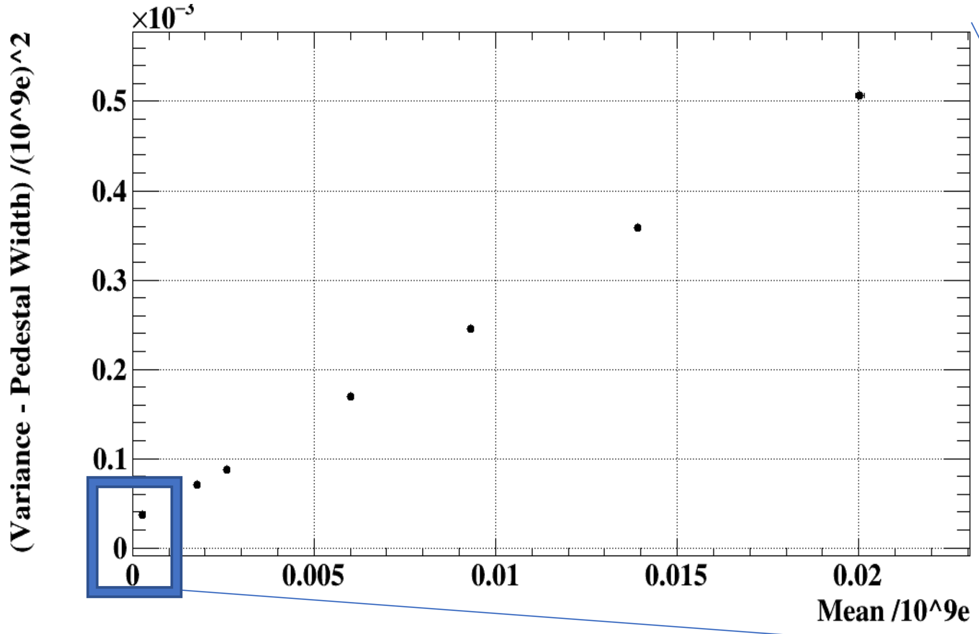
$$\frac{dp}{dx} = \sum_{k=0} GP_{k;\mu,\lambda} \cdot Gauss_{x;k,\sigma_k}$$

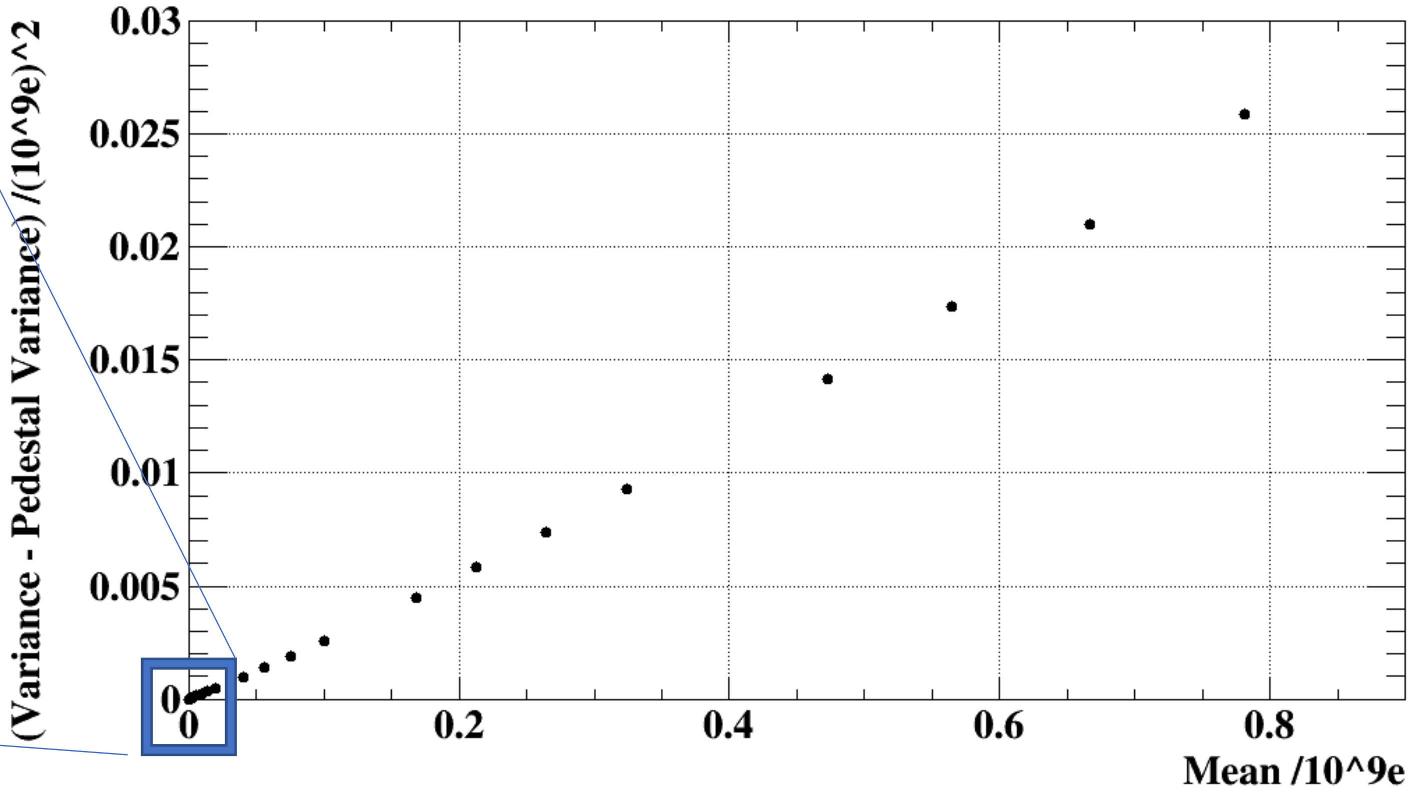
  - $mean - pedestal = \frac{\mu}{1-\lambda} gain$
  - $var - \sigma_0^2 = \left(\frac{gain}{(1-\lambda)^2} + \frac{\sigma_1^2}{gain}\right)(mean - pedestal) \sim \frac{gain}{(1-\lambda)^2}(mean - pedestal)$

Dark Count, Delayed Crosstalk, Afterpulsing

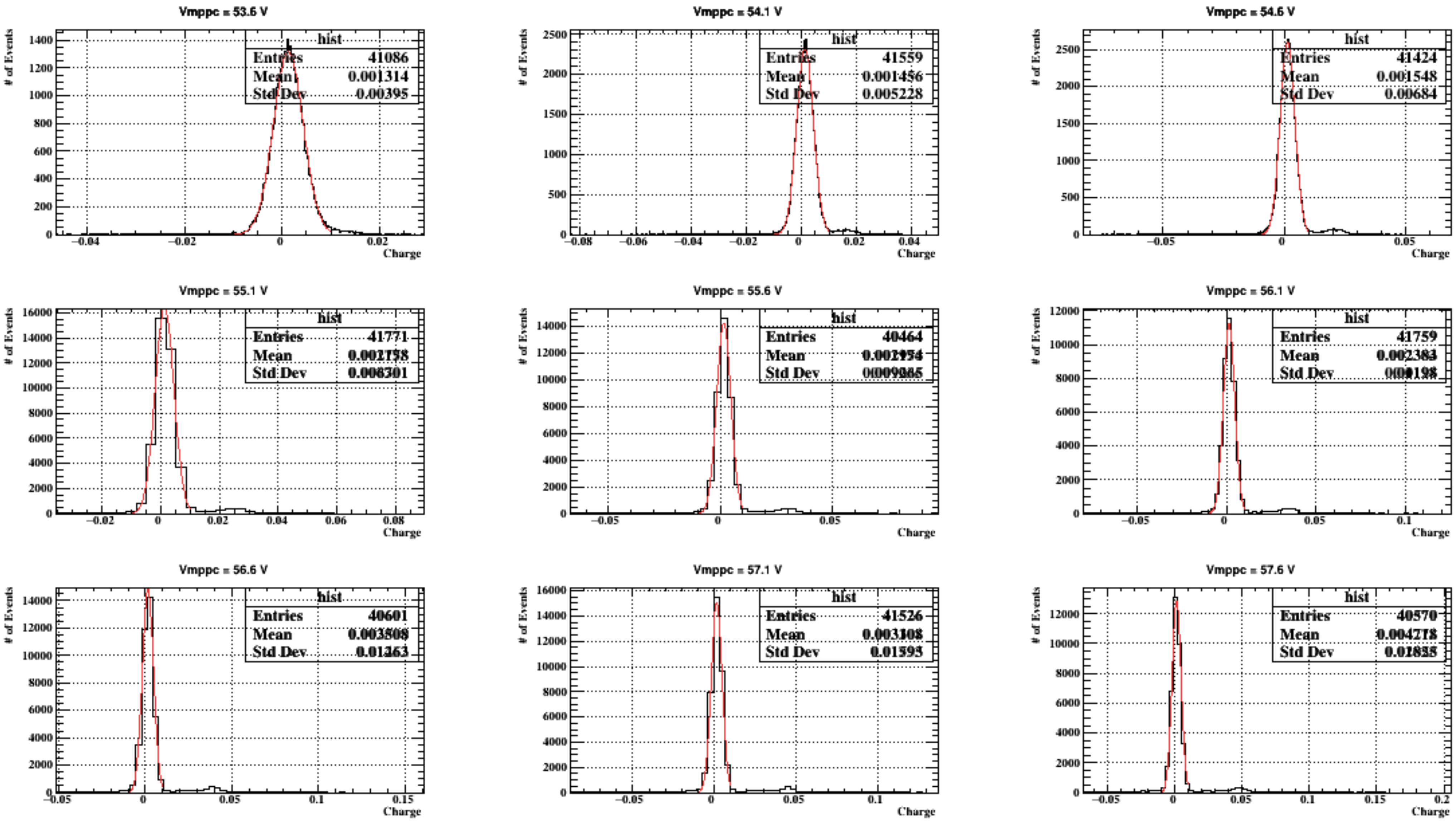
- Dark Count
  - DCR (Dark Count Rate): # of Dark Count in a unit time
  - $p_{DC} = \frac{1}{DCR \cdot IntegrationTime}$
  - $\frac{dp}{dx} = (1 - p_{DC}) GP_0; \mu, \lambda$
    $+ \sum_{k=1} \{(1 - p_{DC}) GP_{k;\mu,\lambda} \cdot Gauss_{x;k,\sigma_k} + p_{DC} GP_{k-1;\mu,\lambda} \cdot Gauss_{x;k-1,\sigma_{k-1}}\}$
- Delayed Crosstalk
  - sometimes come in integration range
  - Charge is same as prompt Crosstalk
- AfterPulsing
  - amplitude is small because under quenching recovery
  - timing follows exponential distribution

Dark Count

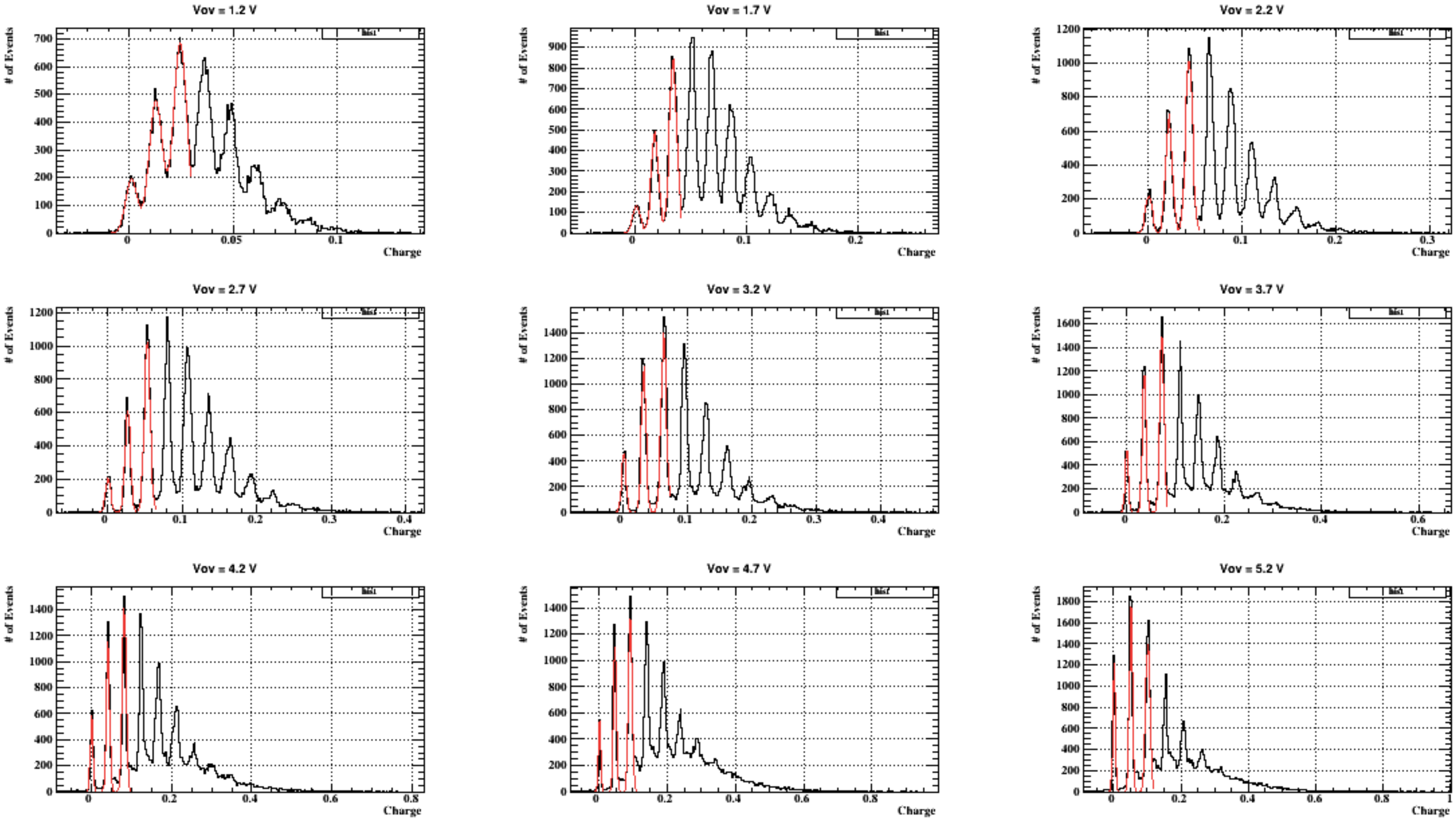

×10⁻³
(Variance - Pedestal Width) /(10^9e)^2
0.5
0.4
0.3
0.2
0.1
0
0
0.005
0.01
0.015
0.02
Mean /10^9e


(Variance - Pedestal Variance) /(10^9e)^2
0.03
0.025
0.02
0.015
0.01
0.005
0
0
0.2
0.4
0.6
0.8
Mean /10^9e

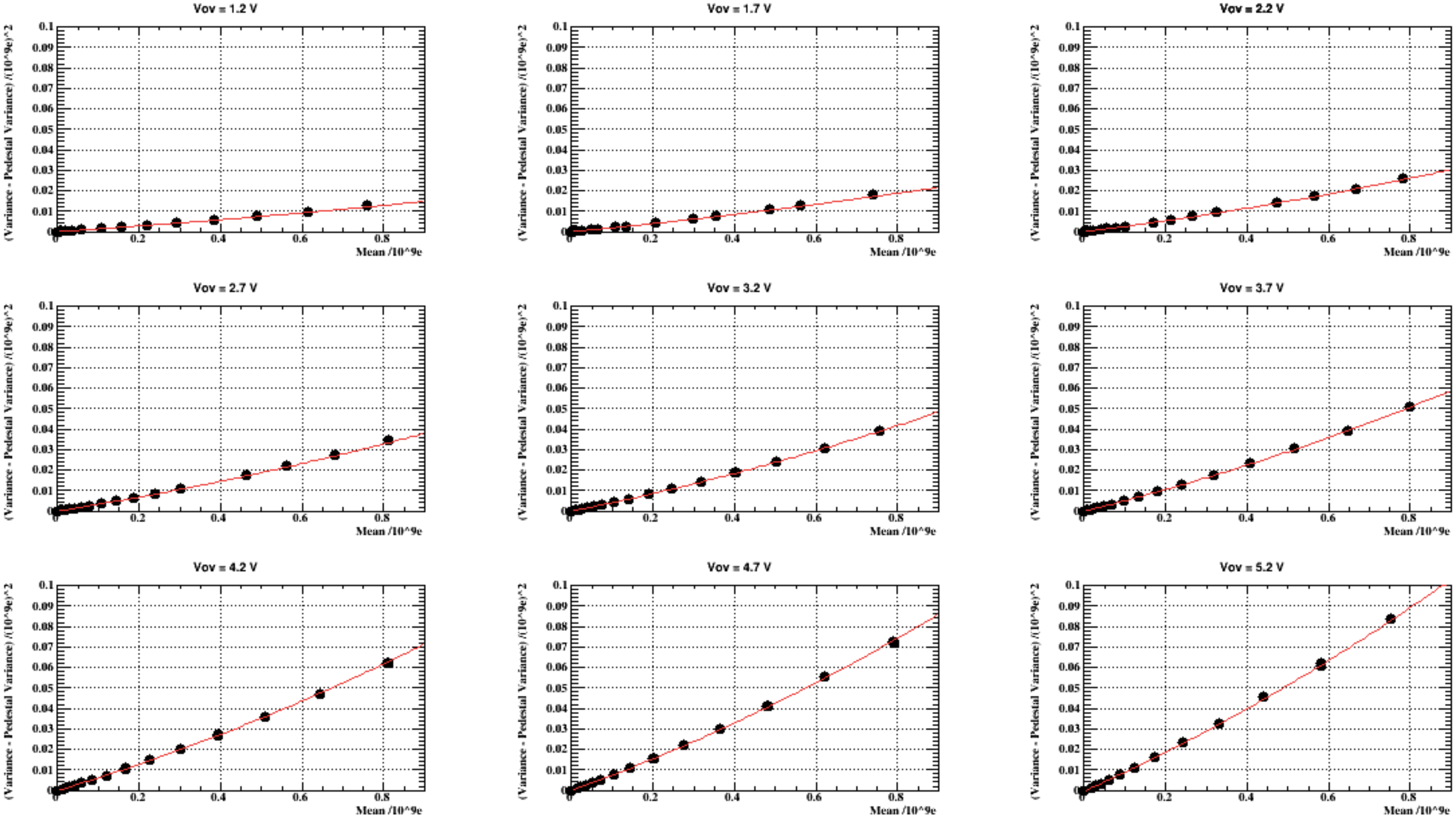
Pedestal Run



Charge Histogram of Weak Light

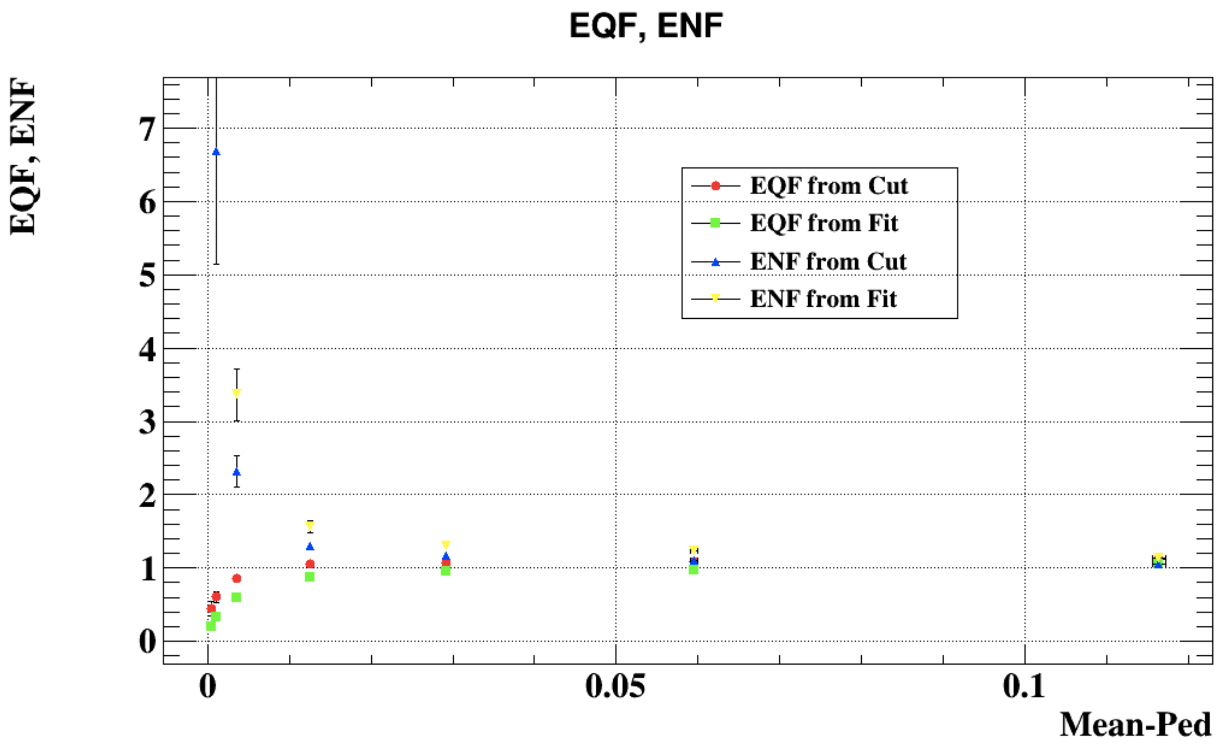


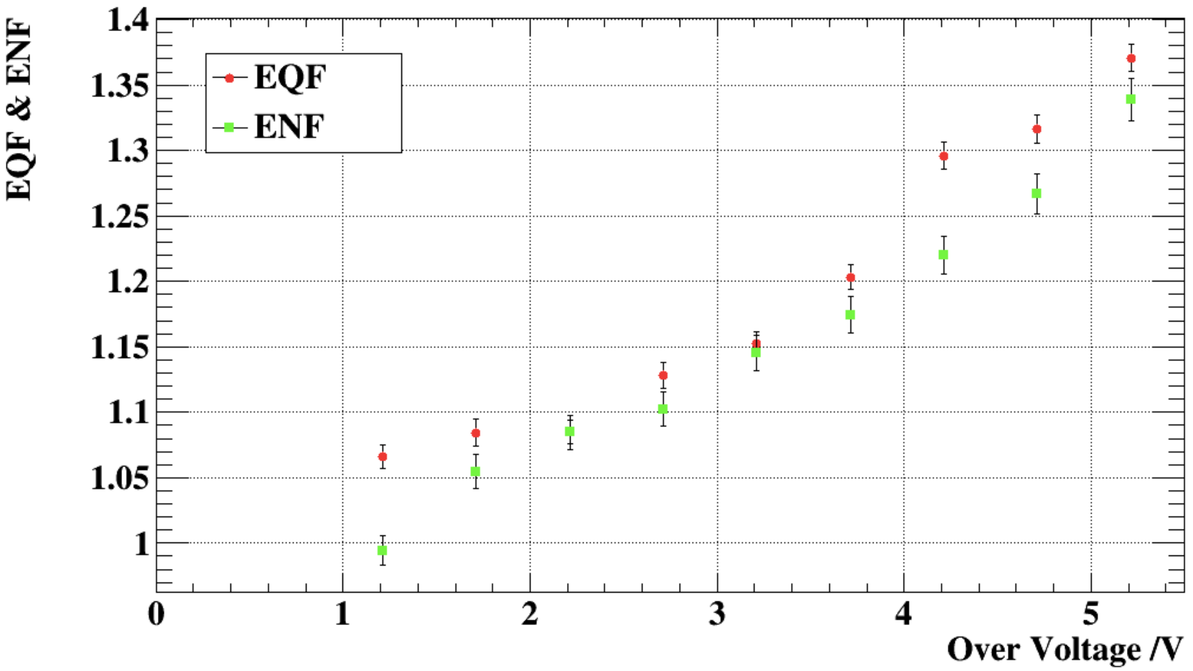
Variance vs Mean



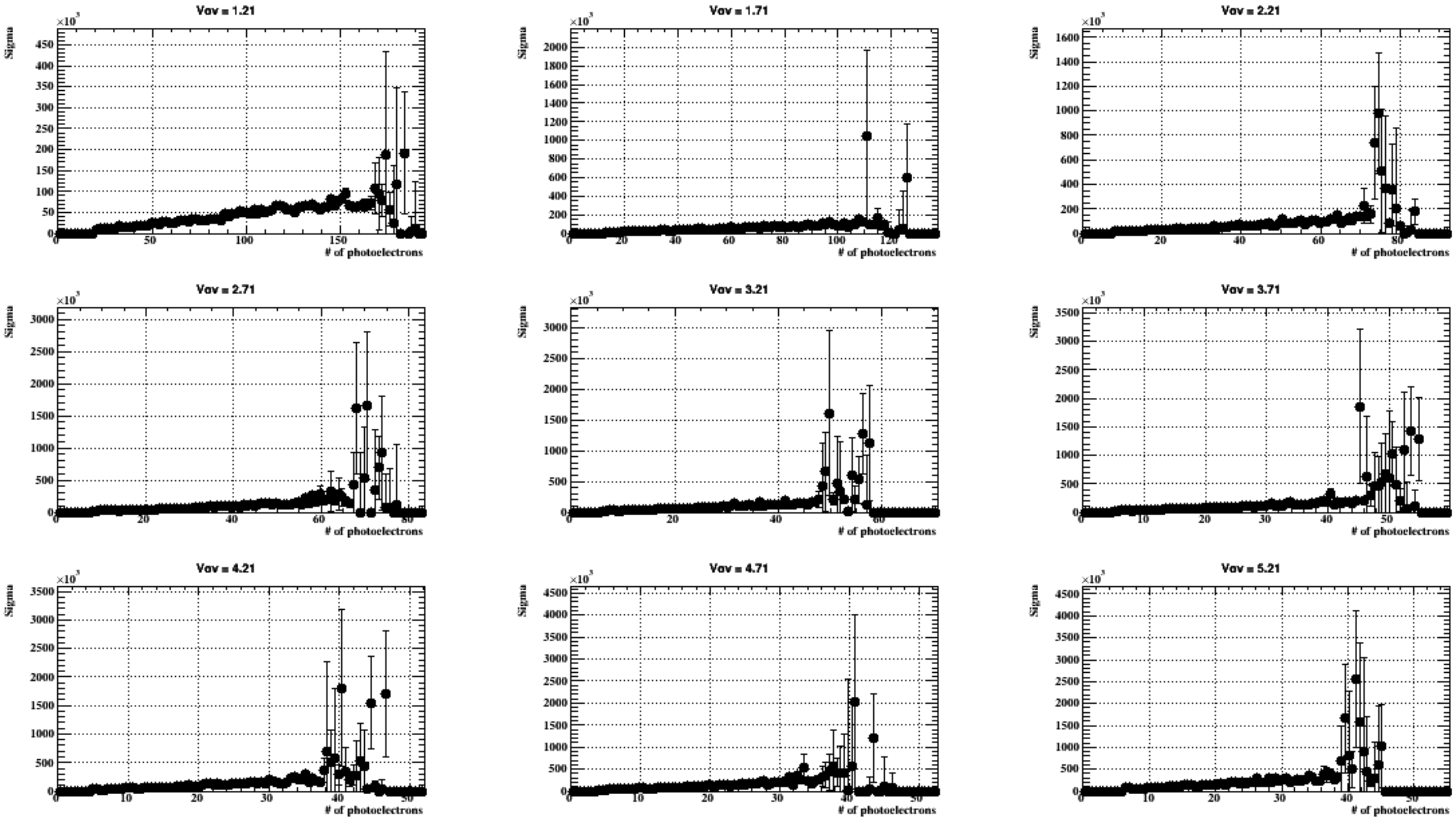
EQF & ENF

- $ENF \coloneqq \mu \dfrac{variance - \sigma_0^2}{(mean - pedestal)^2}$
- $EQF \coloneqq \dfrac{(mean - pedestal)/gain}{\mu}$

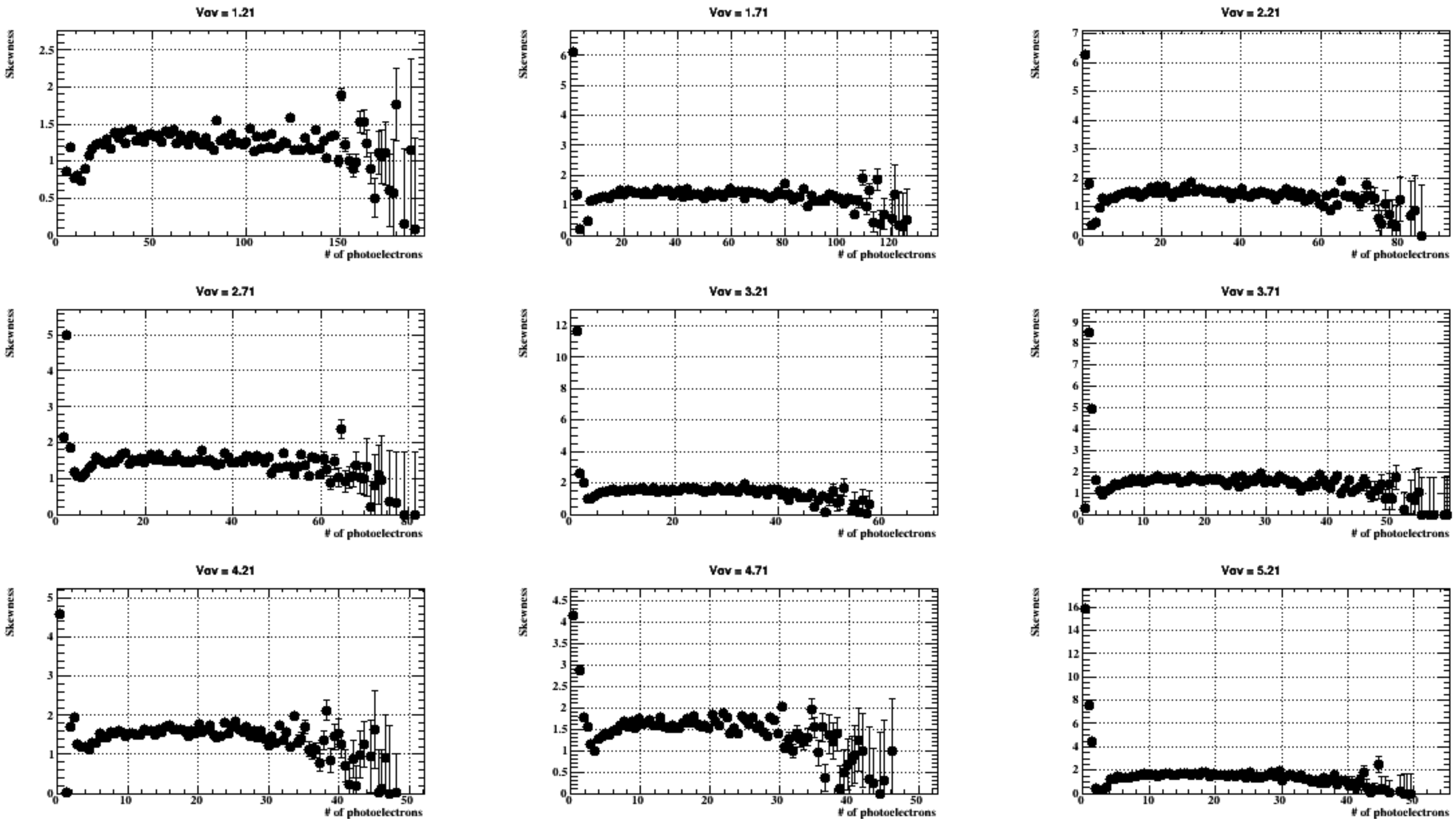




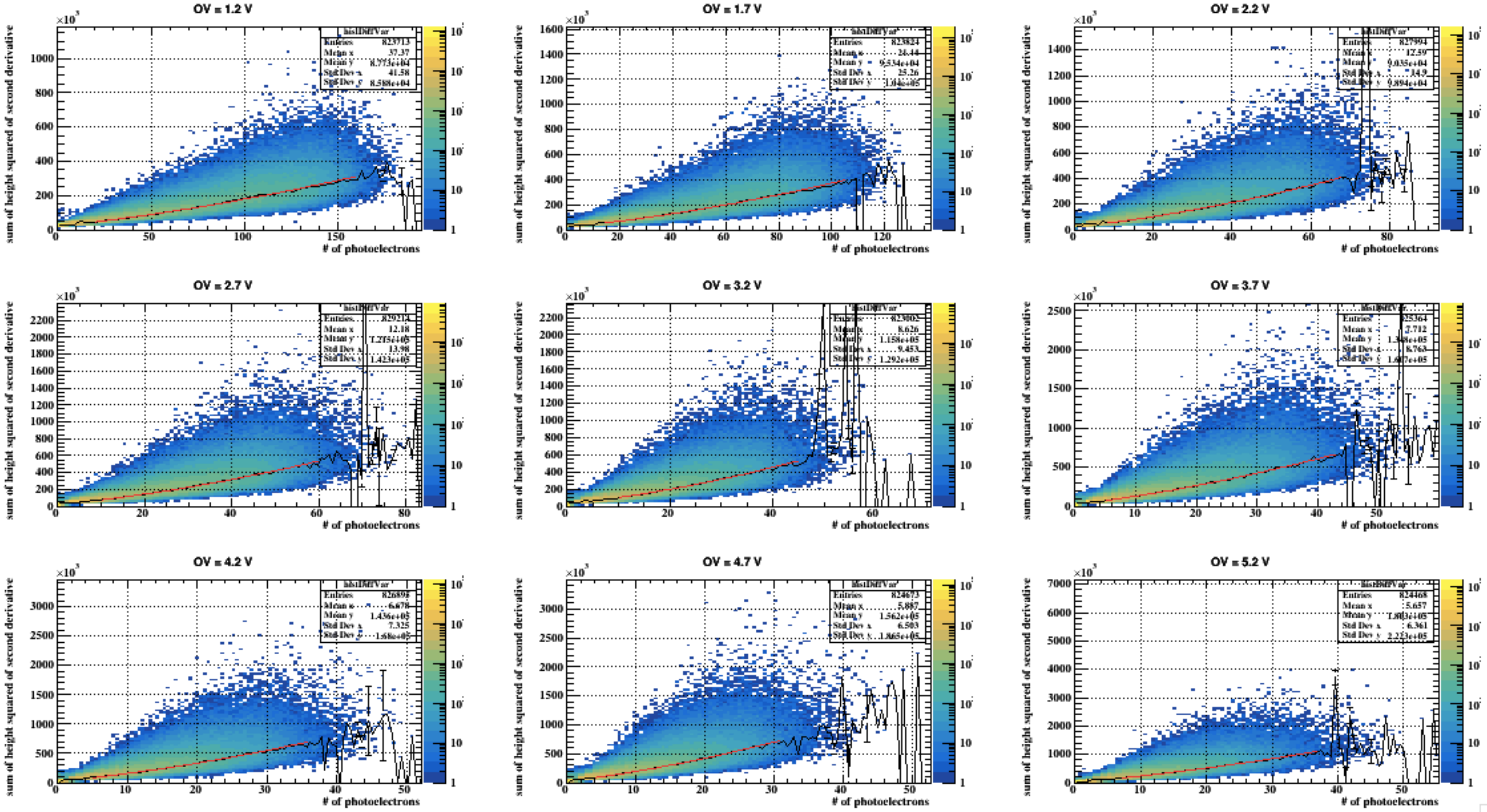
Sigma


Vov = 1.21
Vov = 1.71
Vov = 2.21
Vov = 2.71
Vov = 3.21
Vov = 3.71
Vov = 4.21
Vov = 4.71
Vov = 5.21
Sigma
# of photoelectrons

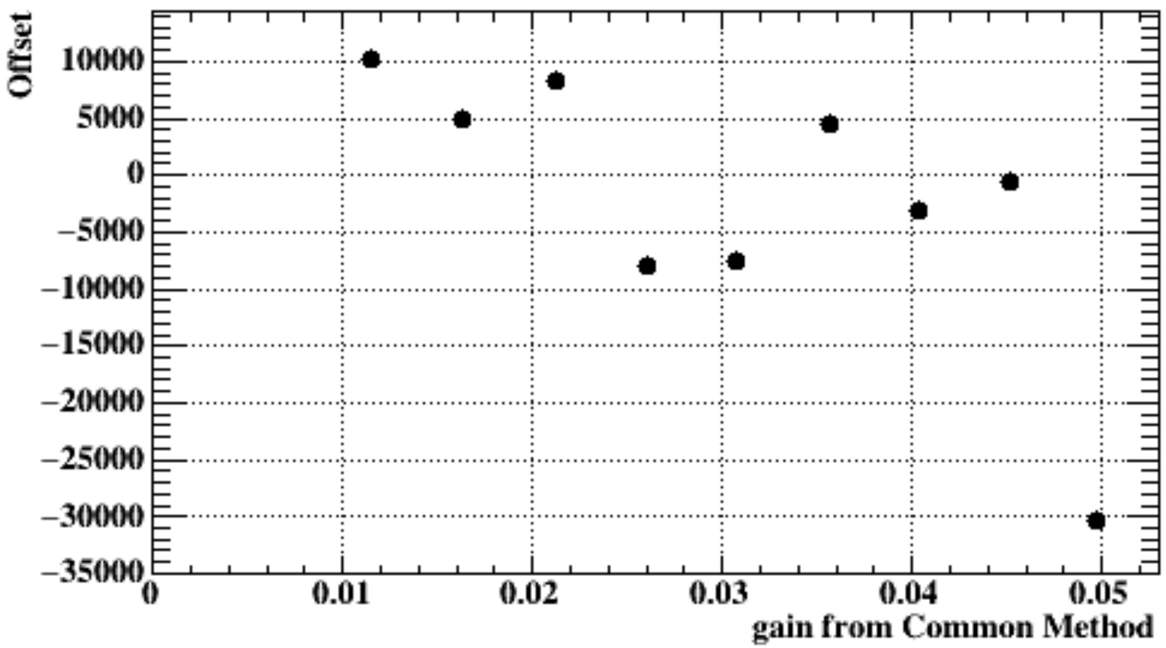
Skewness

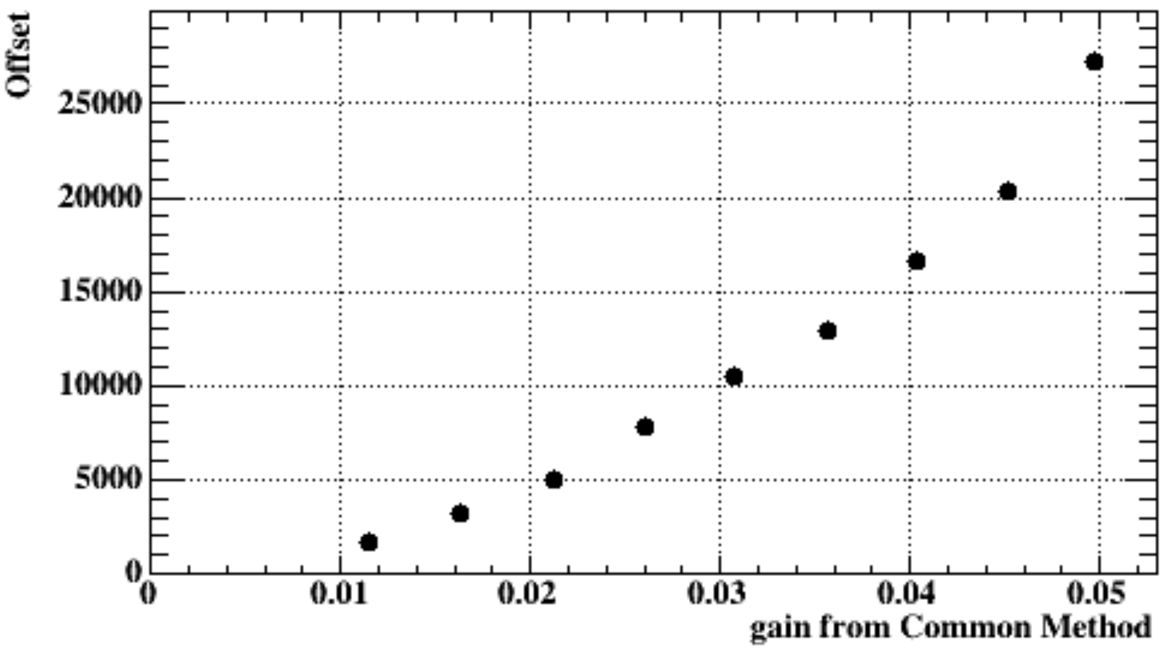

Vov = 1.21
Vov = 1.71
Vov = 2.21
Vov = 2.71
Vov = 3.21
Vov = 3.71
Vov = 4.21
Vov = 4.71
Vov = 5.21
Skewness
# of photoelectrons

Height Squared vs Charge

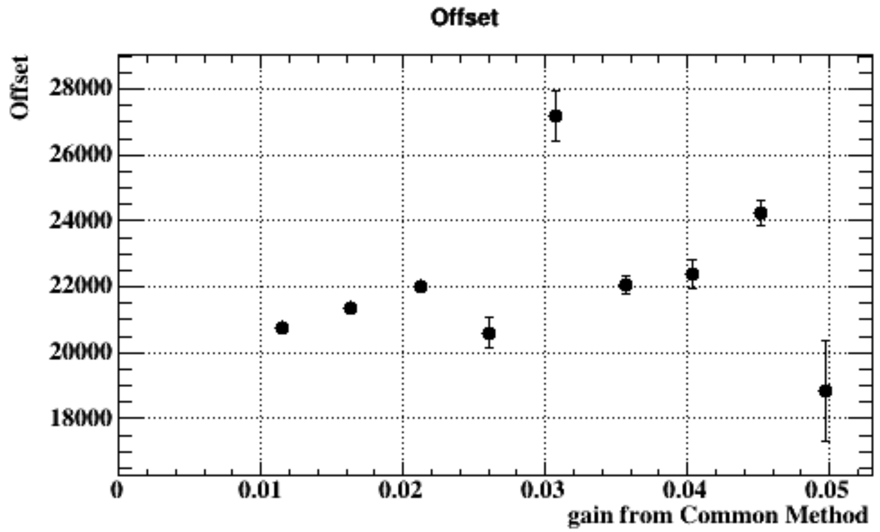


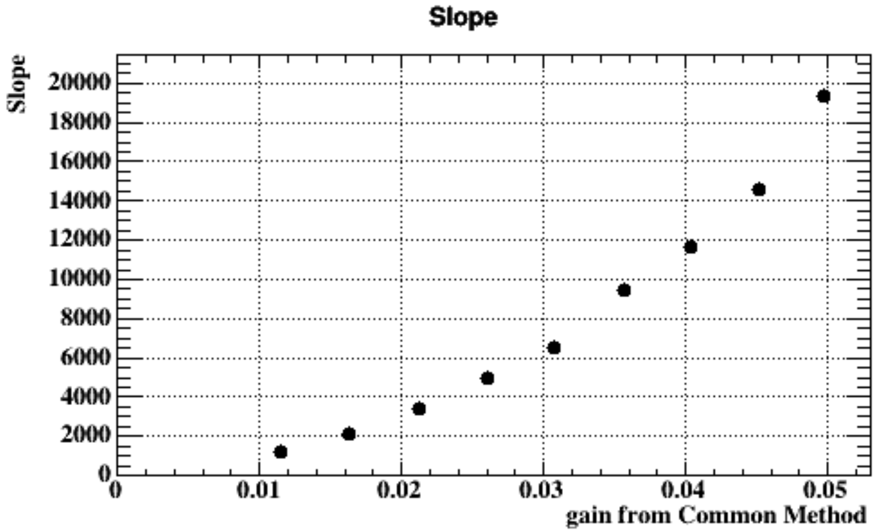
Linear Fitting

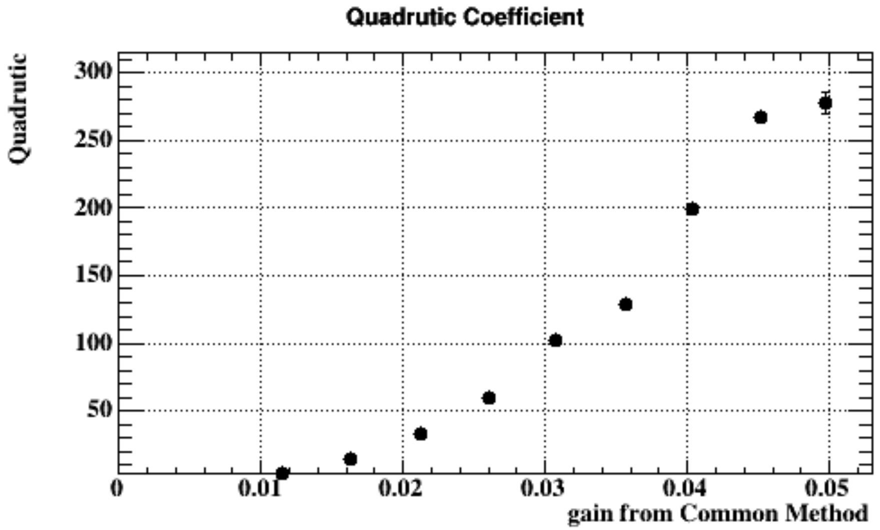




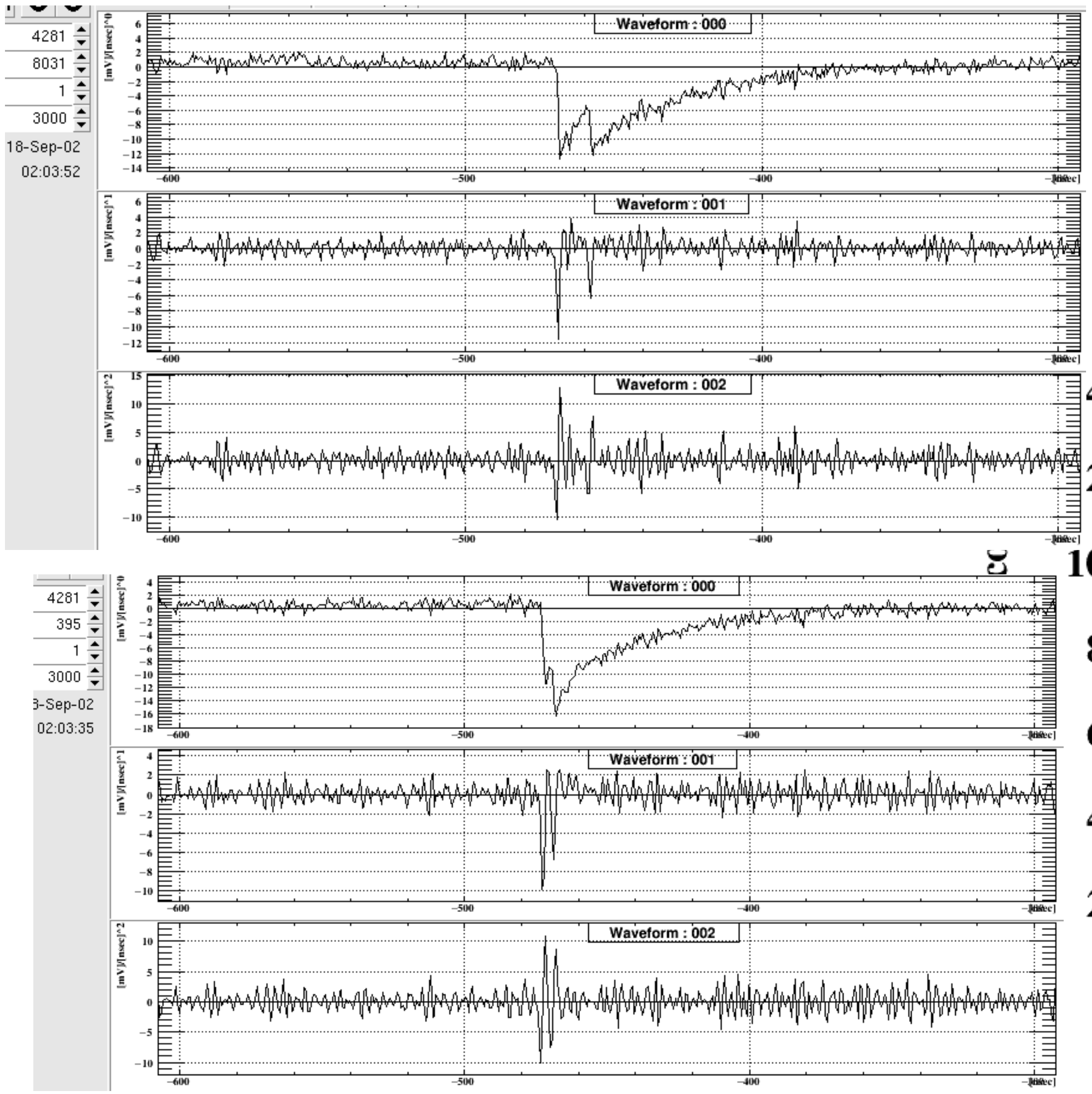
Quadratic Fitting

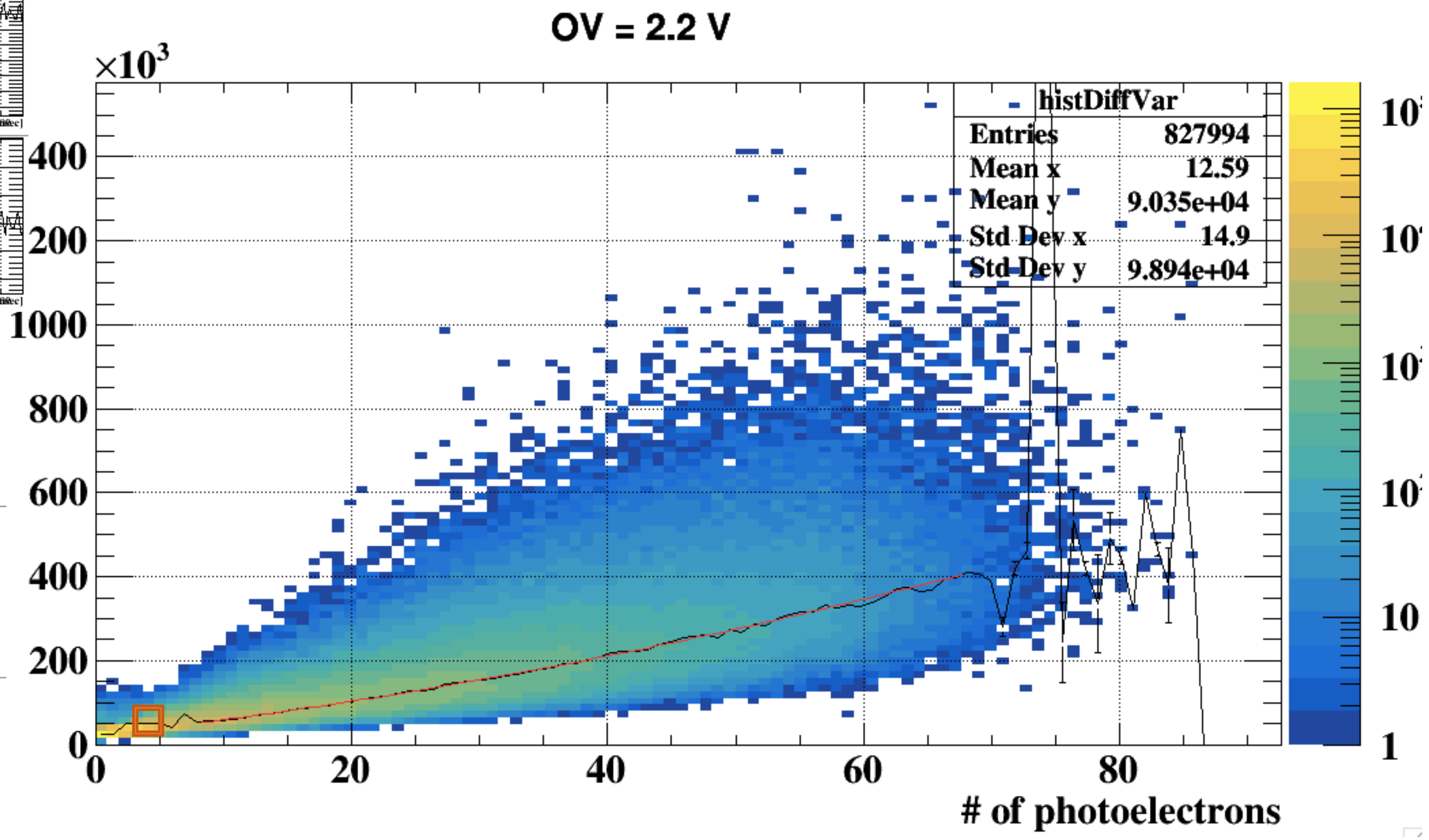




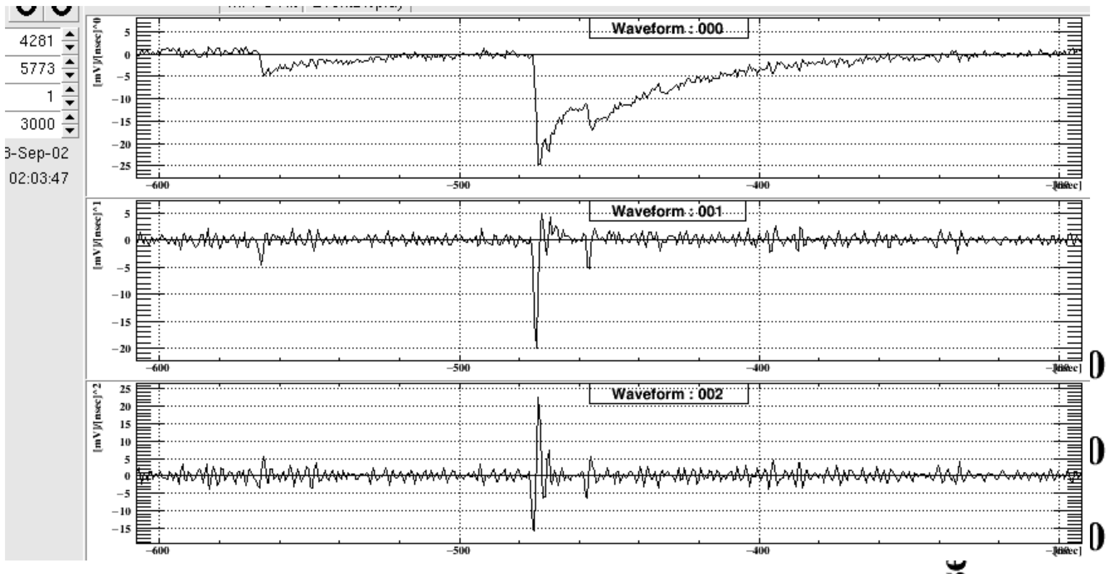


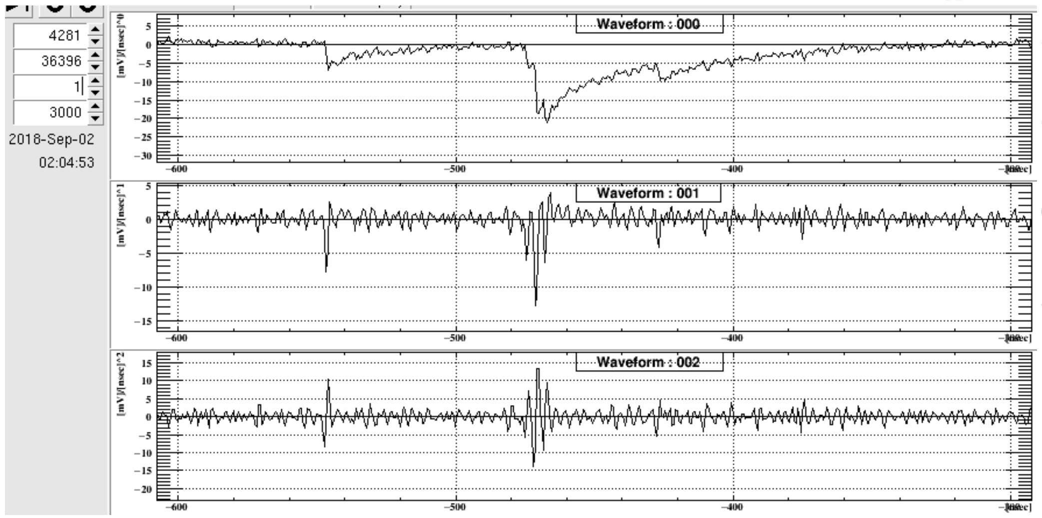
Waveforms

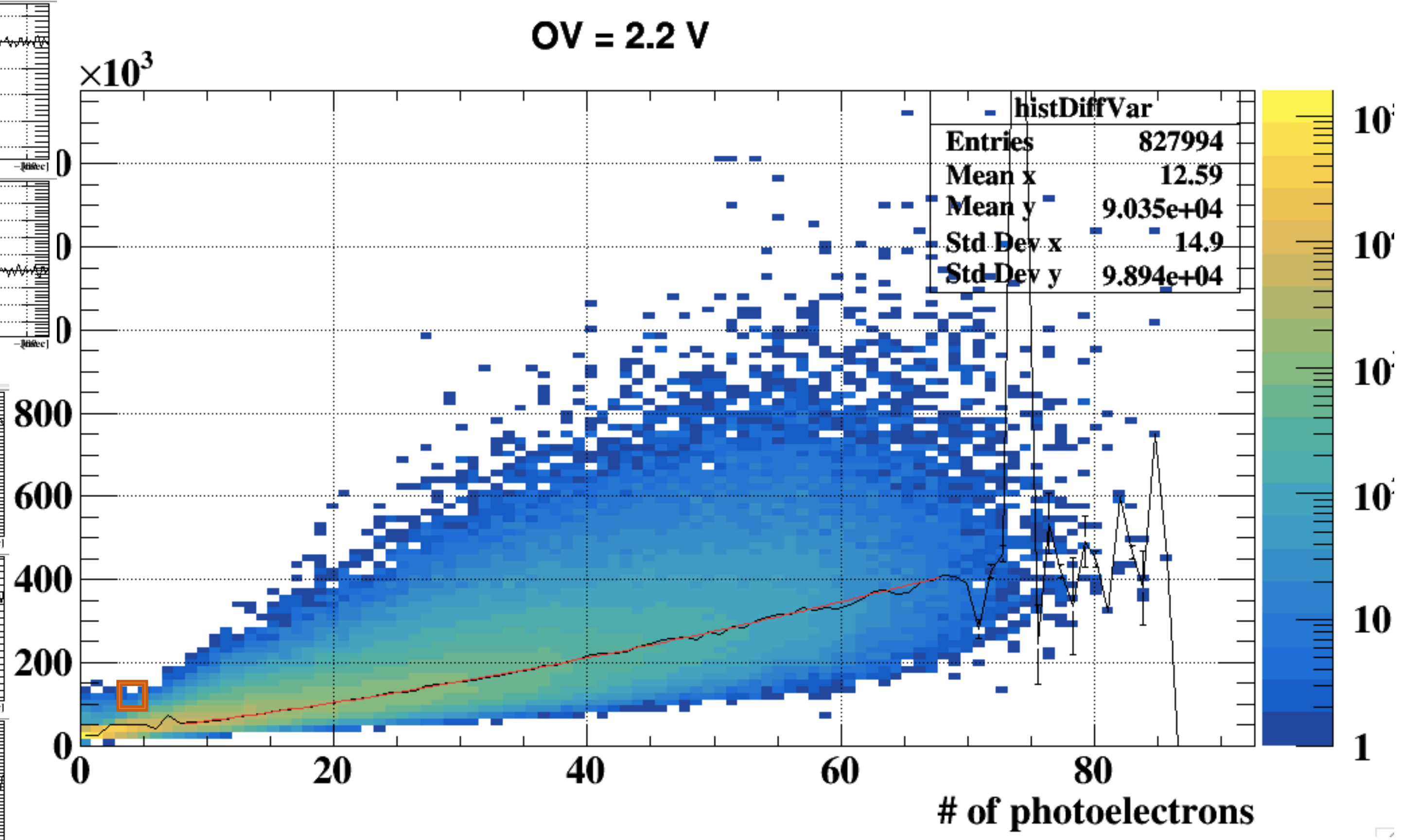

Waveform : 000
Waveform : 001
Waveform : 002
4281
8031
1
3000
18-Sep-02
02:03:52
4281
395
1
3000
02:03:35


OV = 2.2 V
histDiffVar
Entries 827994
Mean x 12.59
Mean y 9.035e+04
Std Dev x 14.9
Std Dev y 9.894e+04
# of photoelectrons

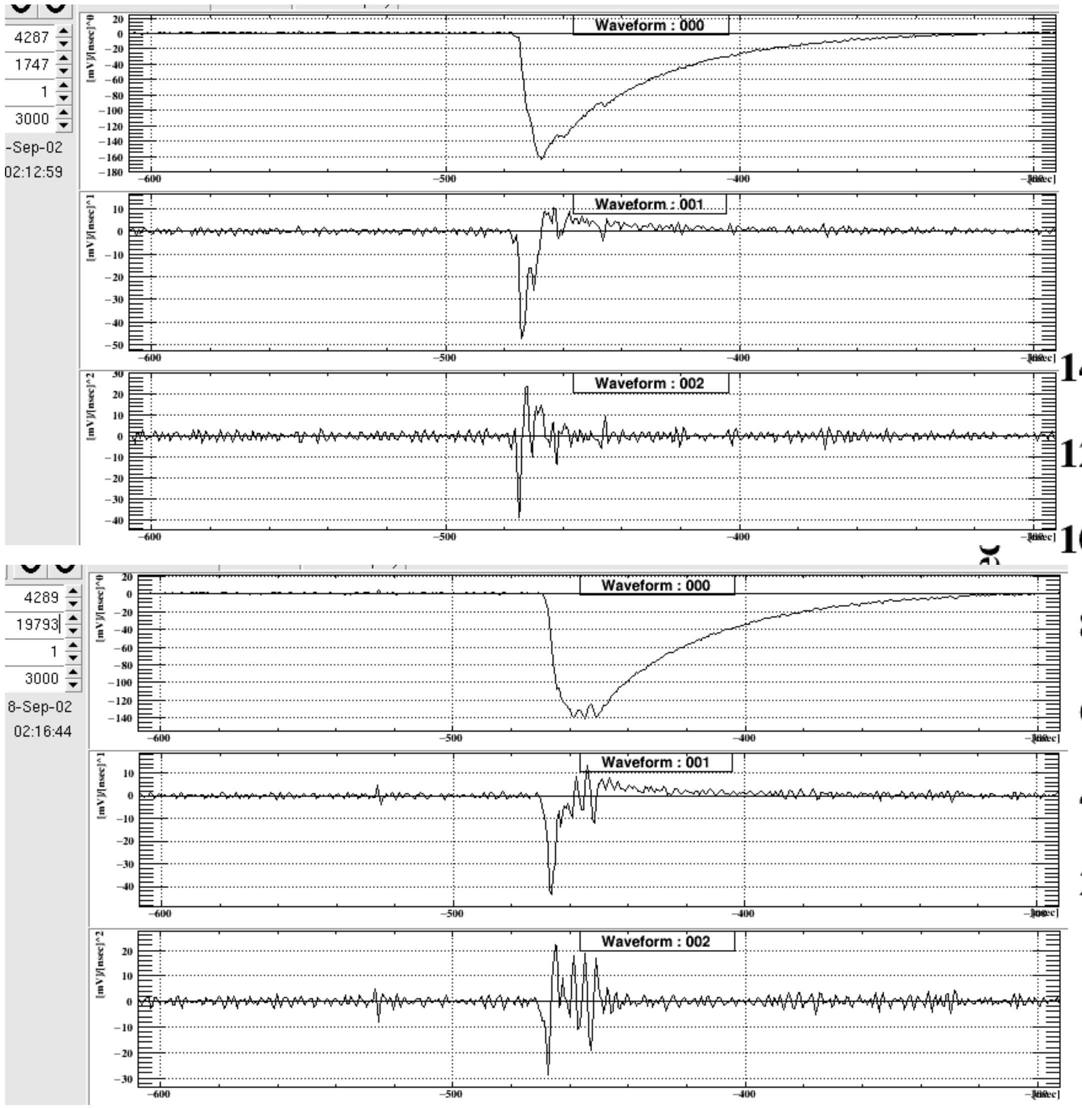
Waveforms

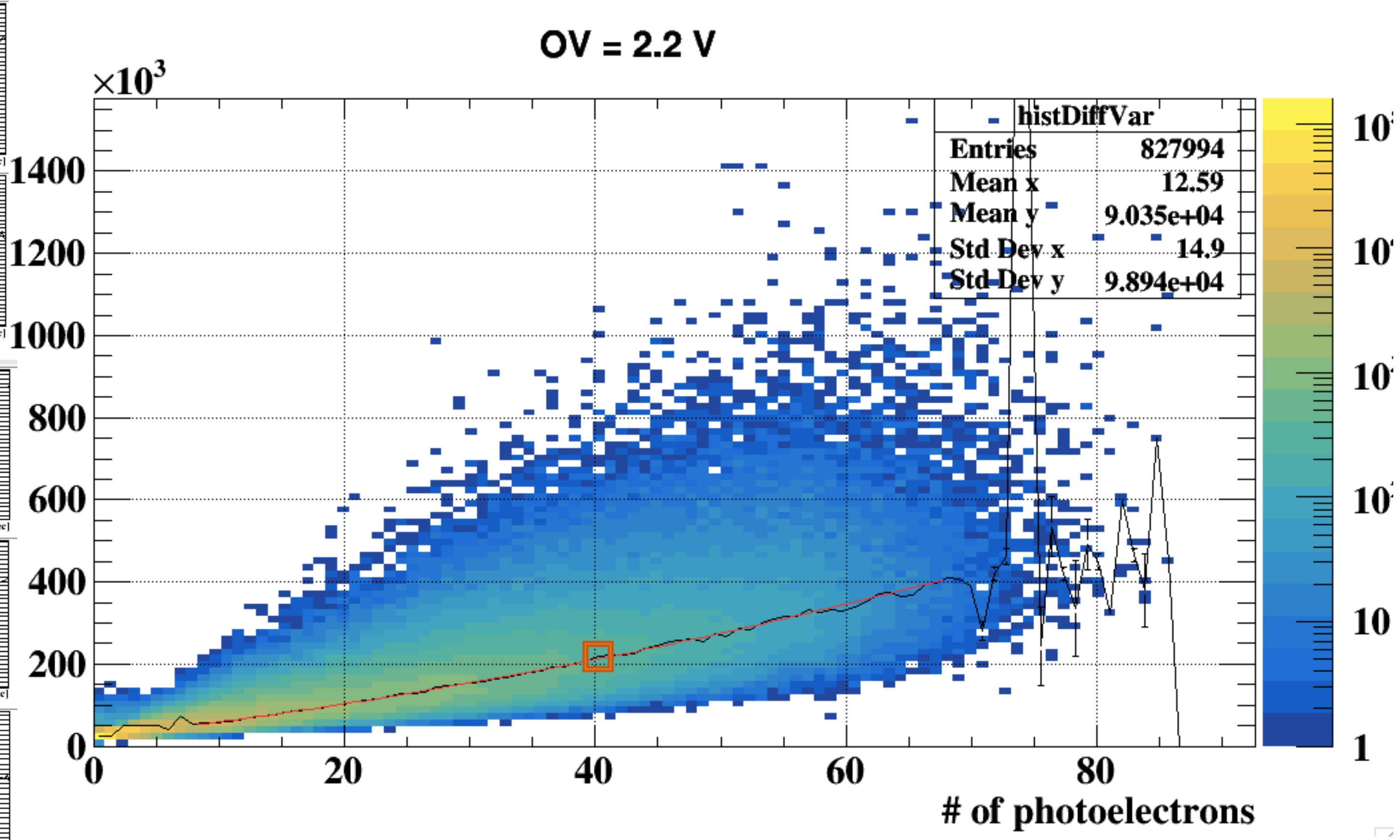

Waveform : 000
Waveform : 001
Waveform : 002
4281
5773
1
3000
02:03:47


Waveform : 000
Waveform : 001
Waveform : 002
4281
36396
1
3000
2018-Sep-02
02:04:53

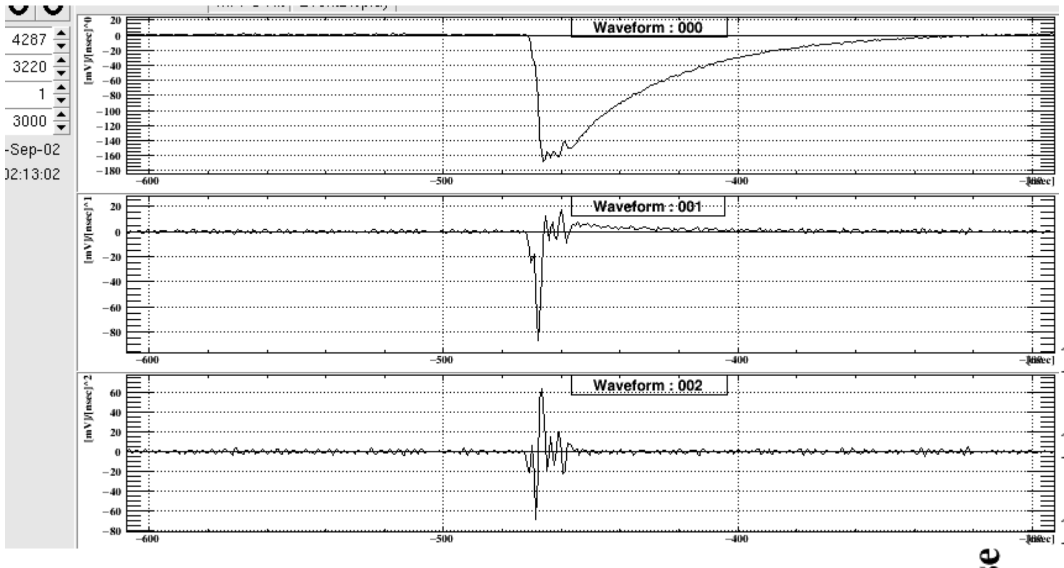

OV = 2.2 V
$\times 10^3$
histDiffVar
Entries 827994
Mean x 12.59
Mean y 9.035e+04
Std Dev x 14.9
Std Dev y 9.894e+04
800
600
400
200
0
0
20
40
60
80
# of photoelectrons
10

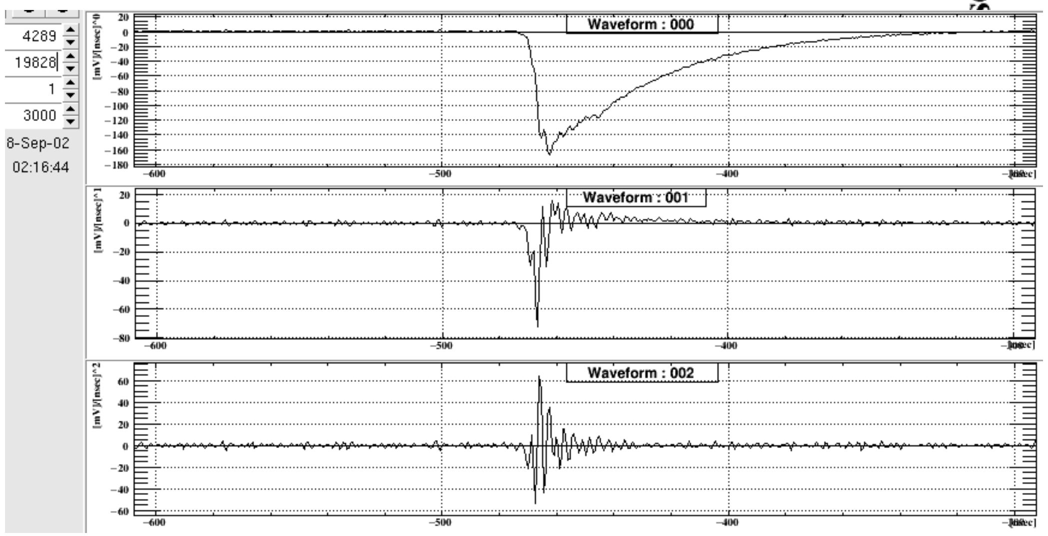
Waveforms

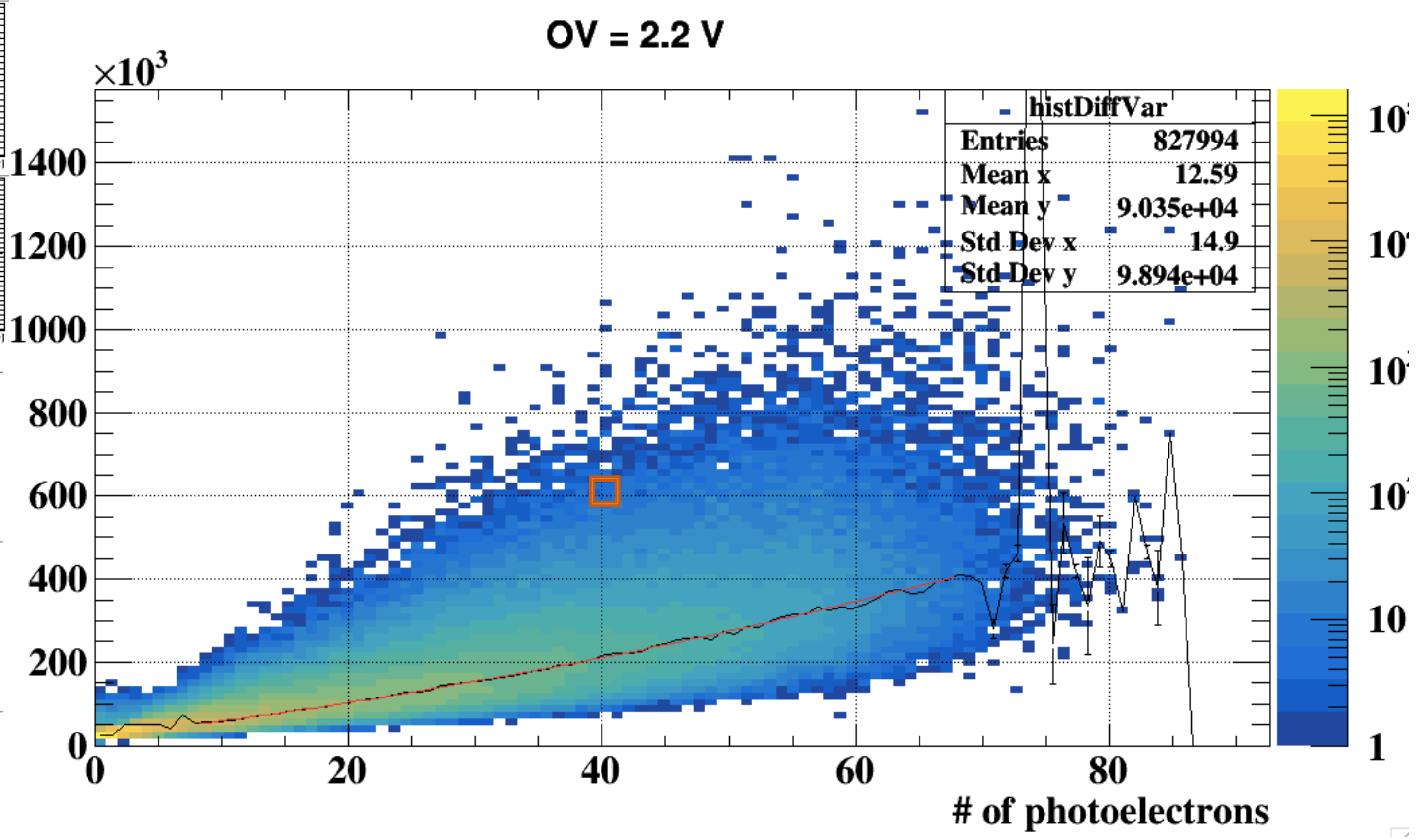

4287
1747
1
3000
-Sep-02
02:12:59
Waveform : 000
Waveform : 001
Waveform : 002
4289
19793
1
3000
8-Sep-02
02:16:44
Waveform : 000
Waveform : 001
Waveform : 002


OV = 2.2 V
histDiffVar
Entries 827994
Mean x 12.59
Mean y 9.035e+04
Std Dev x 14.9
Std Dev y 9.894e+04
# of photoelectrons

Waveforms


4287
3220
1
3000
-Sep-02
02:13:02
Waveform : 000
Waveform : 001
Waveform : 002


4289
19828
1
3000
8-Sep-02
02:16:44
Waveform : 000
Waveform : 001
Waveform : 002


OV = 2.2 V
histDiffVar
Entries 827994
Mean x 12.59
Mean y 9.035e+04
Std Dev x 14.9
Std Dev y 9.894e+04
×10^3
# of photoelectrons