MEG実験におけるミュージ粒子放射崩壊の測定と利用

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ICEPP, the University of Tokyo

内山 雄祐
Introduction

- Studying radiative muon decay (RMD) is important and interesting
  - have to evaluate background rate for $\mu \rightarrow e\gamma$ search.
  - timing calibration tool.
  - internal check of analysis.
  - Michel parameters can be measured.

- We present result of the RMD analysis in the $\mu \rightarrow e\gamma$ search sample collected in 2009 & 2010.
MEG experiment

- Search for the lepton-flavor violating muon decay $\mu \rightarrow e\gamma$
  - Detectors, Beam, DAQ are all optimized to $\mu \rightarrow e\gamma$ detection

$B < 2.4 \times 10^{-12}$
giving stringent constraint on new physics

COBRA positron spectrometer

$\mu \rightarrow e\gamma$

(PRL 107, 171801(2011))
Theory

- SM process RMD is explicitly & accurately calculated in electroweak theory.
- Differential BR of RMD in V-A structure can be written:
  \[ dB(\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu\gamma) = \frac{\alpha}{64\pi^3} \beta dx \frac{dy}{y} d\Omega_e d\Omega_\gamma [F(x, y, d) + \beta P \cos \theta_e G(x, y, d) + P \cos \theta_\gamma H(x, y, d)], \]

- Polarization of decaying muon in MEG target is preserved
  - Measured 0.89±0.04 with Michel spectrum.

Yusuke UCHIYAMA, the University of Tokyo

\[ P = 0.9 \]

E_e > 45 MeV
E_\gamma > 40 MeV

[Graph showing differential branching ratio vs. \( \theta_{e\gamma} \) with \( \theta_{e\gamma} = (\pi - \theta_e) - \theta_\gamma \)]
MEG acceptance & efficiency

- MEG setup limits the phase space able to be measured.
- Direction-match in trigger also deforms RMD spectrum
  - Angle efficiency depends on $E_e$
  - Calculate efficiency table based on MC, and check with measured distribution of accidental background.
Counting RMD events

- Event selection
  - Tracking quality (uncert, $\chi^2$) of track fitting (Kalman filter)
  - Matching with TC hits in space & time
  - Originating at target
  - Cosmic-ray cut in LXe
  - They are identical to those for $\mu \rightarrow e\gamma$ search

- Measure number of RMD with fit to time distribution
  - no assumption on energy and angle distribution
  - high timing resolution of MEG enables it

![Graph showing RMD peak and Accidental BG with N_{\mu\rightarrow e\gamma} = 5749 \pm 197 for RUN2009 and N_{\mu\rightarrow e\gamma} = 7171 \pm 224 for RUN2010]
Projected Distributions

- 2009+2010
- BG are subtracted

Normalized to Michel positrons
Gray bands show systematic (diagonal elements)
BR measurement

- measure BR of RMD for possible largest window: $E_e > 45$, $E_\gamma > 40$ MeV
- Fit to time distribution
  - $45 < E_e < 53$ MeV
  - $40 < E_\gamma < 53$ MeV
  - $|\theta_{e\gamma}| < 0.3$ rad
  - $|\phi_{e\gamma}| < 0.3$ rad
- $N^{\text{obs}} = 12920 \pm 299$ event
- Normalization to Michel
  - in pre-scaled unbiased trigger

$$B(\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma) = \frac{N_{e\nu\gamma}}{N_{\mu} \cdot \epsilon_{e\nu\gamma}} = \frac{N_{e\nu\gamma}}{N_{e\nu\bar{\nu}} \cdot p \cdot (1/f) \cdot (\epsilon_{e\nu\gamma}/\epsilon_{e\nu\bar{\nu}})},$$

$$B(\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma) = (6.03 \pm 0.14 \pm 0.53) \times 10^{-8}$$

for $(E_e > 45, E_\gamma > 40$ MeV),

uncertainties

<table>
<thead>
<tr>
<th>Source</th>
<th>(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photon energy scale</td>
<td>3.4</td>
</tr>
<tr>
<td>Photon response &amp; efficiency curve</td>
<td>2.1</td>
</tr>
<tr>
<td>Positron response &amp; efficiency curve</td>
<td>6.1</td>
</tr>
<tr>
<td>Time response</td>
<td>0.5</td>
</tr>
<tr>
<td>Angle response</td>
<td>&lt; 0.1</td>
</tr>
<tr>
<td>Angle matching efficiency</td>
<td>1.2</td>
</tr>
<tr>
<td>Angle dependence of efficiency</td>
<td>0.6</td>
</tr>
<tr>
<td>Muon polarization</td>
<td>&lt; 0.1</td>
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<tr>
<td>Absolute photon efficiency</td>
<td>2.5</td>
</tr>
<tr>
<td>Absolute trigger efficiency</td>
<td>1.0</td>
</tr>
<tr>
<td>Michel normalization</td>
<td>2.8</td>
</tr>
<tr>
<td>Total systematic</td>
<td>8.5</td>
</tr>
<tr>
<td>Statistical</td>
<td>2.3</td>
</tr>
<tr>
<td>Total (added in quadrature)</td>
<td>8.8</td>
</tr>
</tbody>
</table>
Comparisons

\[ \mathcal{B}^{\text{SM}}(\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma) = 6.15 \times 10^{-8} \]

\[ \frac{\text{BR}_{\text{exp}}}{\text{BR}_{\text{theo}}} = 0.98 \pm 0.09 \]

- Ratio to theory (SM)

No definition of ‘total’ BR (infrared divergent)
BR in limited phase space
Spectrum analysis

- Compare spectrum shape with theory
- Extract \{polarization, normalization\} from RMD

- Fit distribution in multi dimensions
  - binned fit
  - \((E_e, E_\gamma, \theta_{e\gamma}) = 2x2x6, \text{total 24 bins}\)
    - bins in \(\theta_{e\gamma}\) for polarization measurement

- To incorporate correlations of systematic uncertainties among bins, covariance (error) matrix should be used

- Chi-square definition

\[
\chi^2(P_\mu, \alpha) = \sum_{i=1}^{n} \sum_{j=1}^{n} (N_{i}^{obs} - N_{i}^{cal})(V^{-1})_{ij}(N_{j}^{obs} - N_{j}^{cal})
\]

\[N_{i}^{cal} = B_i(P_\mu) \cdot \epsilon_i \cdot \alpha N_\mu \quad (\alpha : \text{normalization scale parameter})\]
Fit result

- no constraint on pol nor norm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi2 / NDF</td>
<td>11.9 / 22</td>
<td>0.541</td>
</tr>
<tr>
<td>Pol</td>
<td>0.70 ± 0.16</td>
<td></td>
</tr>
<tr>
<td>Norm</td>
<td>0.95 ± 0.044</td>
<td></td>
</tr>
</tbody>
</table>

measured value with Michel
Pol 0.89 ± 0.04
Norm 1±0.04
Discussions

- Impact on $\mu \rightarrow e \gamma$ search
  - Powerful internal check of the experiment
    - (calibration, resolutions, efficiencies, normalization, etc.)
  - Estimated $N_{\text{RMD}}$ in $\mu \rightarrow e \gamma$ search before unblinding, and used it in the likelihood as a constraint.
  - Provides a good normalization channel
    - Systematics are independent of Michel channel

- Branching ratio measurement
  - Measurement for largest phase space was given by old experiment (1961)
  - Most precise one has been given by recent result from PIBETA experiment (still preliminary?)
  - MEG gives one for the most stringent window (close to kinematical edge)
    - Relevant to the background in $\mu \rightarrow e \gamma$ search
    - The most sensitive to the deviation from SM
    - Require huge amount of muon decays and precise detectors to measure tiny BR.

- Furthermore, RMD from polarized muon decay
  - No measurement so far. **MEG is a unique experiment.**
  - Provides a possibility of measuring an unmeasured Michel parameter.
RMD rate and parameters

- In general, RMD rate can be written as (Fronsdal&Ueberall, Phys.Rev.113-2(1959)654),

\[ BR(\mu^+ \to \bar{\nu}_\mu e^+ \nu_e \gamma) = F + (1 - \frac{4}{3}\rho)I + \bar{\eta}L \]

\[ + P_\mu \xi \{\beta \cos \theta_e [G - \frac{1}{3}(1 - \frac{4}{3}\delta)J + \kappa M] + \cos \theta_\gamma [H - \frac{1}{3}(1 - \frac{4}{3}\delta)K + \kappa N]\} \]

- \( F,G,H,I,J,K,L,M,N \) are function of (x, y, \( \Theta_{\gamma} \))
  - In V-A, only \( F,G,H \) terms remain.
  - If no polarization, only \( F,I,L \) terms remain.
- \( \rho, \delta, \xi \) are Michel parameters which can be determined by normal Michel decay measurements.
- \( \bar{\eta}, \kappa \) are the new Michel parameters which can only be determined by RMD
  - \( \bar{\eta} : \) spectrum shape
    - \( \bar{\eta} = (|g_{RL}|^2 + |g_{LR}|^2) + \frac{1}{8}(|g_{LR}^s + 2g_{LR}^T|^2 + |g_{RL}^s + 2g_{RL}^T|^2) + 2(|g_{LR}^T|^2 + |g_{RL}^T|^2) \geq 0. \)
  - \( \kappa : \) asymmetry in polarized muon decay
  - If assume \( \rho = \delta = \frac{3}{4}, \xi = 1, \)
    - \( \bar{\eta} = \kappa = \frac{1}{4} (1 - \xi'), \) 0 \( \leq \bar{\eta} = \kappa \leq \frac{1}{2} \)
    - equivalent to the longitudinal polarization of positron in muon decay

<table>
<thead>
<tr>
<th>V-A</th>
<th></th>
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<tbody>
<tr>
<td>( \rho )</td>
<td>( \frac{3}{4} )</td>
</tr>
<tr>
<td>( \delta )</td>
<td>( \frac{3}{4} )</td>
</tr>
<tr>
<td>( \xi )</td>
<td>1</td>
</tr>
<tr>
<td>( \xi' )</td>
<td>1</td>
</tr>
<tr>
<td>( \bar{\eta} )</td>
<td>0</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>0</td>
</tr>
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</table>
Current status

**$\eta$ PARAMETER**

$(V-A)$ theory predicts $\eta = 0$. $\eta$ affects spectrum of radiative muon decay.

<table>
<thead>
<tr>
<th>VALUE</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>CHG</th>
<th>COMMENT</th>
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<tbody>
<tr>
<td>0.02 ± 0.08</td>
<td>OUR AVERAGE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>−0.014±0.090</td>
<td>EICHENBER... 84 ELEC +</td>
<td>$\rho$ free</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+0.09 ± 0.14</td>
<td>BOGART 67 CNTR +</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>−0.035±0.098</td>
<td>EICHENBER... 84 ELEC +</td>
<td>$\rho=0.75$ assumed</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- recent measurement by PIBETA
  - preliminary result in a dissertation & some conf. proceedings.
  - $\eta = -0.084 \pm 0.050 \pm 0.034$ ($\rho = 0.75$)
  - $\eta < 0.033$ (68% C.L.)
  - They are going to finalize result & publish it by the summer 2012

- No measurement of $\kappa$ parameter
Conclusion

- Analyzed RMD from polarized muon decay in MEG data
- Measure BR for MEG acceptance range
  - $\text{BR}(E_e > 45, E_\gamma > 40 \text{MeV}) = (6.03 \pm 0.14 \pm 0.53) \times 10^{-8}$
  - $\text{BR}^{\text{exp}}/\text{BR}^{\text{theo}} = 0.98 \pm 0.09$
  - Good agreement with SM
- Spectrum analysis (fit)
  - Good agreement with SM ($\chi^2/\text{NDF} = 11.9/22$)
  - Extract polarization and normalization
    - RMD only: Polarization: $0.70 \pm 0.16$
      Normalization factor: $0.95 \pm 0.043$ (relative to one from Michel)
    - Consistent result with prediction
      - less powerful for polarization measurement
      - Normalization uncertainty goes down to 4%
- Possibility of measuring Michel parameters. Next step.
Polarization

- surface muons are fully polarized
- depolarization effects
  - beam angular acceptance: 2.5%
  - multiple scattering in production target: <0.5%
  - contribution from ‘cloud muon’: 4.5%
  - E-field in beamline (separator): 0.7%
  - in plastic target (moderation, muonium formation): 3%

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**Table 1.** Values of muon residual polarization at zero field for various targets

<table>
<thead>
<tr>
<th>Material</th>
<th>$\mu^+$ polarization (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium</td>
<td>(100.2 ± 2)</td>
</tr>
<tr>
<td>Carbon</td>
<td>(100.0 ± 2)</td>
</tr>
<tr>
<td>Calcium</td>
<td>(99.0 ± 2)</td>
</tr>
<tr>
<td>Freon 11</td>
<td>(79.5 ± 3)</td>
</tr>
<tr>
<td>Freon 13</td>
<td>(78.5 ± 3.5)</td>
</tr>
<tr>
<td>Polyethylene</td>
<td>(67.1 ± 2)</td>
</tr>
<tr>
<td>Teflon</td>
<td>(62.0 ± 2)</td>
</tr>
<tr>
<td>Emulsion</td>
<td>(35.8 ± 2)</td>
</tr>
<tr>
<td>Scintillator</td>
<td>(20.4 ± 2)</td>
</tr>
<tr>
<td>Sulphur</td>
<td>(10.0 ± 3)</td>
</tr>
</tbody>
</table>

A.Buhler et al. Nouvo Cimento **39**, 824 (1965)

---

Polyethylene

MEG magnet center

1.3 T

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A.Buhler et al. Nouvo Cimento **39**, 812 (1965)
Polarization measurement

- Measure polarization from angular distribution of Michel positrons
- Two-dimensional fit to $E_e$ vs $\theta_e$ distribution gives $P = 0.90 \pm 0.04$
Radiative correction to RMD

Fig. 1. Types of Feynman diagrams for radiative muon decay with one-loop RC.

- Papers

- Now under evaluation for MEG case

<table>
<thead>
<tr>
<th></th>
<th>$10^2\sigma_1$</th>
<th>$10^2\sigma_2$</th>
<th>$10^2\theta$</th>
<th>$10^2\delta_1$</th>
<th>$10^2\delta_{\gamma\gamma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\xi = 0$</td>
<td>$\xi = 0.5$</td>
<td>$\xi = -0.5$</td>
<td>$\xi = 0$</td>
<td>$\xi = 0.5$</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>-1.2</td>
<td>-1.0</td>
<td>-1.3</td>
</tr>
<tr>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
<td>-3.7</td>
<td>-3.0</td>
<td>-4.0</td>
</tr>
<tr>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
<td>-6.1</td>
<td>-5.0</td>
<td>-6.7</td>
</tr>
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<td>6.0</td>
<td>6.0</td>
<td>3.0</td>
<td>-10.0</td>
<td>-8.6</td>
<td>-10.8</td>
</tr>
<tr>
<td>3.0</td>
<td>3.0</td>
<td>5.9</td>
<td>4.4</td>
<td>3.8</td>
<td>4.9</td>
</tr>
<tr>
<td>4.0</td>
<td>4.0</td>
<td>3.0</td>
<td>-6.0</td>
<td>-5.0</td>
<td>-6.5</td>
</tr>
</tbody>
</table>

Table 1: Numerical estimates for the corrections $\delta_1$ and $\delta_{\gamma\gamma}$ versus $\sigma_1, \sigma_2, \theta, \xi$.

+ real (soft or hard) photon emission

Yusuke UCHIYAMA, the University of Tokyo
Chisquare

- Define chisquare

\[ \chi^2(P, \alpha) = \sum_{i=1}^{n} \sum_{j=1}^{n} (N_{i,\text{obs}} - N_{i,\text{cal}})(V^{-1})_{ij}(N_{j,\text{obs}} - N_{j,\text{cal}}) \]

\[ N_{i,\text{cal}} = B_i(P, \mu) \cdot \epsilon_i \cdot \alpha N_{\mu} \quad (\alpha : \text{normalization scale parameter}) \]

\[ V = V_{\text{stat}} + \Sigma V_{\text{sys}} : \text{covariance matrix} \]

- See the deviation of \( N_{i,\text{cal}} \) when each systematic parameter is varied by 1 standard deviation.

\[ V_{\text{stat}} = \begin{pmatrix} \sigma_1^2 & 0 & \cdots \\ 0 & \sigma_2^2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \quad V_{\text{sys}} = \begin{pmatrix} \delta_1 \delta_1 & \delta_1 \delta_2 & \cdots \\ \delta_1 \delta_2 & \delta_2^2 & \cdots \end{pmatrix} \]
Error matrix

- correlation matrix
• For the energy range, $\pi - \Theta_{e\gamma} < 0.048$ rad is allowed.
**Fit result 2**

- Constrain pol & norm

Pol  \(-0.88 \pm 0.04\)
Norm  \(0.96 \pm 0.038\)

\(\text{Chi}^2 / \text{NDF} = 13.6 / 22 = 0.617\)
From previous studies

- \( BR(\mu^+ \rightarrow \bar{\nu}_\mu e^+ \nu_e \gamma) = F + \left(1 - \frac{4}{3}\rho\right)I + \eta L \)
  \[ + P_\mu \left\{ \beta \cos \theta_e \left[ G \frac{1}{3}(1 - \frac{4}{3}\delta)J + \kappa M \right] + \cos \theta_\gamma \left[ H \frac{1}{3}(1 - \frac{4}{3}\delta)K + \kappa N \right] \right\} \]

- \((L/F)(x,y)\) plot show the ‘sensitivity’ of \( \bar{\eta} \) measurement.
  - the closer to back-to-back and the higher \( x \), the larger contribution of \( L \) term relative to SM (F term).
  - Of course, the statistics also decrease as closing to kinematic edge.
  - MEG data is suitable
    - High statistics data for back-to-back & large \( x \).

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Yusuke UCHIYAMA, the University of Tokyo
### Michel parameters in PDG

#### $\rho$ parameter

(V–A) theory predicts $\rho = 0.75$.

<table>
<thead>
<tr>
<th>Value</th>
<th>EVTS</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>CHG COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7503 ± 0.0004</td>
<td>OUR AVERAGE</td>
<td>MACDONALD 08</td>
<td>TWST</td>
<td>Surface $\mu^+$</td>
</tr>
<tr>
<td>0.75014 ± 0.00017</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75080 ± 0.00032</td>
<td>6G</td>
<td>MUSSER 05</td>
<td>TWST</td>
<td>Surface $\mu^+$</td>
</tr>
<tr>
<td>0.7518</td>
<td>DERENZO 69</td>
<td>RVUE</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### $\delta$ parameter

(V–A) theory predicts $\delta = 0.75$.

<table>
<thead>
<tr>
<th>Value</th>
<th>EVTS</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>CHG COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7504 ± 0.0006</td>
<td>OUR AVERAGE</td>
<td>MACDONALD 08</td>
<td>TWST</td>
<td>Surface $\mu^+$</td>
</tr>
<tr>
<td>0.75067 ± 0.00030</td>
<td>6G</td>
<td>GAPONENKO 05</td>
<td>TWST</td>
<td>Surface $\mu^+$</td>
</tr>
<tr>
<td>0.74964</td>
<td>BALKE 88</td>
<td>SPEC</td>
<td>SIN, $\pi$ decay in flight</td>
<td></td>
</tr>
<tr>
<td>0.7486 ± 0.0026</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### $|\langle \xi \rangle \times (\mu \text{ LONGITUDINAL POLARIZATION})|$ parameter

(V–A) theory predicts $\xi = 1$, longitudinal polarization = 1.

<table>
<thead>
<tr>
<th>Value</th>
<th>EVTS</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>CHG COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0007 ± 0.0035</td>
<td>OUR AVERAGE</td>
<td>JAMIESON 06</td>
<td>TWST</td>
<td>Surface $\mu^+$ beam</td>
</tr>
<tr>
<td>1.0003 ± 0.0006</td>
<td>6G</td>
<td>BELTRAMI 87</td>
<td>CNTR</td>
<td>SIN, $\pi$ decay in flight</td>
</tr>
<tr>
<td>1.0027 ± 0.0079</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### $\xi'$ = LONGITUDINAL POLARIZATION OF $e^+$

(V–A) theory predicts the longitudinal polarization = ±1 for $e^\pm$, respectively. We have flipped the sign for $e^-$ so our programs can average.

<table>
<thead>
<tr>
<th>Value</th>
<th>EVTS</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>CHG COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00 ± 0.04</td>
<td>OUR AVERAGE</td>
<td>BURKARD 85</td>
<td>CNTR</td>
<td>Bhabha + annihil</td>
</tr>
<tr>
<td>0.998 ± 0.045</td>
<td>1M</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.89 ± 0.28</td>
<td>SCHWARTZ 67</td>
<td>OSPK</td>
<td>Moller scattering</td>
<td></td>
</tr>
<tr>
<td>0.94 ± 0.38</td>
<td>BLOOM 64</td>
<td>CNTR</td>
<td>Brems. transmiss.</td>
<td></td>
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<tr>
<td>1.04 ± 0.18</td>
<td>DUCLOS 64</td>
<td>CNTR</td>
<td>Bhabha scattering</td>
<td></td>
</tr>
<tr>
<td>1.05 ± 0.30</td>
<td>BUHLER 63</td>
<td>CNTR</td>
<td>Annihilation</td>
<td></td>
</tr>
</tbody>
</table>

Normal Michel parameters are measured very precisely by dedicated experiments.

No chance to improve with MEG data.